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PROCEDURES FOR ANALYZING INTERFERENCE CAUSED BY SPREAD-SPECTRUM--ETC(U)
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LEVEL II (11)

PROCEDURES FOR ANALYZING INTERFERENCE CAUSED BY SPREAD-SPECTRUM SIGNALS

IIT Research Institute
Under Contract to
DEPARTMENT OF DEFENSE
Electromagnetic Compatibility Analysis Center
Annapolis, Maryland 21402

February 1978

FINAL REPORT

Approved for public release; distribution unlimited.

78 07 27 036

ESD-TR-77-003

This report was prepared by the IIT Research Institute under Contract F-19628-78-C-0006 with the Electronic Systems Division of the Air Force Systems Command for operation of the DoD Electromagnetic Compatibility Analysis Center, Annapolis, Maryland.

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19 **REPORT DOCUMENTATION PAGE**

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER 18 ESD-TR-77-003	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) 6 PROCEDURES FOR ANALYZING INTERFERENCE CAUSED BY SPREAD-SPECTRUM SIGNALS. [redacted]		5. TYPE OF REPORT & PERIOD COVERED 9 FINAL rept., 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) 10 Paul Newhouse		8. CONTRACT OR GRANT NUMBER(s) 15 F 19628-78-C-0006
9. PERFORMING ORGANIZATION NAME AND ADDRESS DoD Electromagnetic Compatibility Analysis Center, North Severn, Annapolis, MD 21402		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 12 190p.
11. CONTROLLING OFFICE NAME AND ADDRESS 16 649E		12. REPORT DATE 11 Feb[redacted]78 13. NUMBER OF PAGES 197
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

UNLIMITED

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

ELECTROMAGNETIC COMPATIBILITY ANALYSIS
SPREAD SPECTRUM
INTERFERENCE-TO-NOISE RATIO
SIGNAL-TO-INTERFERENCE RATIO

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A mathematical procedure is given for determining whether a spread-spectrum signal will cause an unacceptable interference condition in a victim receiver. The procedure involves calculating either the interference-to-noise ratio or the signal-to-interference ratio at the IF-amplifier output and comparing the ratio with an appropriate threshold value. The front end and IF amplifier of the victim receiver are treated as a bandpass filter and non linear processes are not considered.

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PREFACE

The Electromagnetic Compatibility Analysis Center (ECAC) is a Department of Defense facility, established to provide advice and assistance on electromagnetic compatibility matters to the Secretary of Defense, the Joint Chiefs of Staff, the military departments and other DoD components. The center, located at North Severn, Annapolis, Maryland 21402, is under executive control of the Assistant Secretary of Defense for Communication, Command, Control, and Intelligence and the Chairman, Joint Chiefs of Staff, or their designees, who jointly provide policy guidance, assign projects, and establish priorities. ECAC functions under the direction of the Secretary of the Air Force and the management and technical direction of the Center are provided by military and civil service personnel. The technical operations function is provided through an Air Force sponsored contract with the IIT Research Institute (IITRI).

This report was prepared as part of AF Project 649E under Contract F-19628-78-C-0006 by the staff of the IIT Research Institute at the Department of Defense Electromagnetic Compatibility Analysis Center.

To the extent possible, all abbreviations and symbols used in this report are taken from American Standard Y10.19 (1967) "Units Used in Electrical Science and Electrical Engineering" issued by the USA Standards Institute.

Users of this report are invited to submit comments that would be useful in revising or adding to this material to the Director, ECAC, North Severn, Annapolis, Maryland 21402, Attention ACL.

EXECUTIVE SUMMARY

Analytical procedures are presented for predicting adjacent-signal interference when conventional receiving systems are subjected to offending signals from spread-spectrum (SS) transmitters. The procedures, which are generally applicable to all types of SS signals and to narrowband and wideband receivers, entail three major steps, namely:

1. Predicting the nature of the response waveform produced at the output of the IF amplifier in the victim receiver.
2. Calculating the interference-to-noise ratio (INR) of the response waveform.
3. Comparing the calculated value of INR with an appropriate threshold value (INR_t); if the INR should be less than INR_t, a compatible condition is predicted.

The procedures rely heavily on the use of plots of the spectral characteristics of the offending SS transmitter and the victim receiver, i.e., the emission spectrum and the receiver selectivity. These plots can be either taken from measurements or from calculations that use formulas given in the report.

The accuracy of the predictions depends largely on the accuracy of the parameter values supplied as inputs to the procedures. These parameters include: the SS transmitter power, signaling rate, type of modulation, and antenna gain; the victim receiver noise figure, bandwidth and antenna gain; and the basic radio-transmission loss, which is often referred to as the propagation path loss.

EXECUTIVE SUMMARY (Continued)

Section 2, which is written in handbook style, contains the formulas and an explanation of the parameters that are used in performing steps 1 and 2 listed above. Guidelines are given for selecting an appropriate value for the threshold INR_t, which is used in step 3.

Tutorial material, detailed explanations of the formulas, and several examples of calculations are given in appendixes.

The procedures given here can be applied in various ways, including; predicting adjacent-signal interference, studying parameter tradeoffs and sensitivities, and plotting frequency/distance-separation tradeoff curves that are used to determine the constraints that must be imposed to achieve electromagnetic compatibility between an offending transmitter and a potential victim receiver. The procedures have been incorporated in FDRCAL, a computer program used at ECAC to calculate and plot frequency/distance separation curves.

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GLOSSARY OF TERMS AND SYMBOLS

Spread Spectrum

Dixon[1] defines a spread-spectrum (SS) system as one in which the transmitted signal is spread over a wide frequency band, much wider, in fact, than the minimum bandwidth required to transmit the information being sent. Some signal or operation other than the information being sent is used in broadbanding (or spreading) the transmitted signal.

Wideband FM, however, does not conform to the second part of the above definition and therefore, is not considered to be an SS technique.

Categories of SS Techniques

The various techniques considered here are placed into the following categories, which are similar to those used by Dixon.

1. *Direct sequence (DS)*. This technique utilizes modulation of a carrier by a digital code sequence whose chip rate is much higher than the information bandwidth.

2. *Frequency Hopping (FH)*. For this technique the carrier frequency is shifted in discrete increments in a pattern dictated by a code sequence.

3. *Pulsed FM (PFM)*. The FM carrier is swept, or rapidly stepped, over a wide band during a given pulse interval. In one application, this process is performed with a polychromatic delay line, which is a surface acoustic wave (SAW) filter.

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[1]Dixon, R. C., *Spread Spectrum Systems*, John Wiley & Sons, New York 1976.

4. *Hybrids*. These techniques employ frequency hopping combined with direct-sequence (FH/DS), or with pulsed FM (FM/PFM).

* * *

Symbols and Abbreviations

Note: All symbols and abbreviations are defined where they first appear in the text; those that appear throughout the text are also defined below.

		Page
B_c	- chip rate, megachips/s (Mc/s).	50
B_F	- 3 dB bandwidth of transmitter RF filter (MHz).	54
B_h	- frequency hopping rate, megahops/seconds, (Mh/s).	52
B_i	- bandwidth (MHz), or bit rate (Mb/s) of the information or baseband signal.	52
B_r	- 3 dB bandwidth of the IF amplifier of the victim receiver (MHz).	49
B_t	- bandwidth of SS signal (MHz), defined by Equation 6.	55
B_3	- 3 dB bandwidth of pulsed FM type of SS signal (MHz).	50
C	- allowable increase in the total receiver noise power with respect to N_r; also, the allowed decrease in the receiver signal-to-noise ratio (dB).	62
C_{wb}	- a calculated constant (TABLE 9).	58
c_1	- a constant that defines the point of demarcation between two particular signal/bandwidth conditions.	56

		Page
chip	- smallest interval of commonality in the SS signal, also referred to as a symbol in a radiated signal.	50
DS	- direct sequence.	xi
$E\{\Delta f\}$	- spectral power density function of filtered SS signal or emission spectrum, in dB.[a]	15
$\tilde{E}\{\Delta f\}$	- a function that bounds $E_t\{\Delta f\}$.[a]	15
FH	- frequency-hopping.	xi
FH/DS	- frequency-hopping/direct-sequence.	xii
FH/PFM	- frequency-hopping/pulsed FM.	xii
F_j	- the jth frequency in a frequency-hopped SS signal (MHz).	52
F_n	- the transmitter "noise floor" level referenced to the peak of the transmitter emission spectrum (dB).	56
F_o	- center of passband of transmitter filter (MHz).	57
F_r	- noise figure of the victim receiver; includes ambient as well as receiver thermal noise (dB).	9
f_r	- tuned frequency of victim receiver.	56
f_t	- nominal carrier frequency of SS signal (MHz).	56
Δf	- frequency separation $(f_t - f_r)$, between SS signal and the tuned frequency of the victim receiver (MHz).	56
Δf_j	- frequency separation $(F_j - f_r)$, between the jth frequency of a frequency-hopped SS signal and the tuned frequency of the victim receiver (MHz).	57
Δf_o	- frequency separation $(F_o - f_r)$, between the center of the passband of transmitter filter and tuned frequency of victim receiver (MHz).	57

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[a]An additional subscript is used to denote the type of signal: (t) for DS, (i) for information or baseband, (f) for frequency-hopping, and (h) for FH/DS. See APPENDIX F.

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$F\{\Delta f\}$	- spectral power density function of SS signal[a] (dB).	144
$\tilde{F}\{\Delta f\}$	- a function that bounds $F\{\Delta f\}$, (dB). See Note a.	144
$F_\delta\{\Delta f\}$	- a special spectral function that is used when $B_t/B_r < 1$[a] (dB).	144
$\tilde{F}_\delta\{\Delta f\}$	- a function that bounds $F_\delta\{\Delta f\}$[a] (dB).	144
G_t	- antenna gain of SS transmitter (dBi).	9
G_r	- antenna gain of the victim receiver (dBi).	9
INR	- interference-to-noise (power) ratio at the output of the IF amplifier (dB).	9
INR_i	- interference-to-noise (power) ratio at the input of the receiver (dB).	9
INR_t	- interference-to-noise threshold at the output of the IF amplifier (dB).	62
L_p	- radio transmission loss between the source antenna and the victim-receiver antenna (dB).	9
MSK	- minimum-frequency-shift keying.	146
n	- number of bits in the shift register that is used to generate a maximal-length sequence.	75
N_c	- the total number of chips in the code.	50

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[a]An additional subscript is used to denote the type of signal (t) for DS, (i) for information or baseband, (f) for frequency-hopping, and (h) for FH/DS. See APPENDIX F.

		Page
N/C	- transmitter noise power in 1 Hz bandwidth nomalized to the carrier power (dBm in 1 Hz); a measure of performance of a transmitting device.	56
N_f	- number of frequencies employed in a frequency-hopping SS system.	52
N_r	- noise power in the IF bandwidth of the victim receiver (dBm).	9
OFR	- off-frequency rejection (dB).	89
OQPSK	- offset-quadrature-phase-shift keyed.	145
OTR	- on-tune rejection (dB).	85
PF	- impulse peaking factor; the ratio of the highest to the average INR of the peaks of impulsive interference (dB).	62
P_i, p_i	- power of the SS signal at the input of a bandpass filter (expressed in dBm, or mW).	85
P_o, p_o	- power of the response waveform of a bandpass filter (expressed in dBm, or mW).	85
PSK	- phase-shift keyed.	144
P_t, p_t	- power of SS signal at the transmitter (expressed in dBm, or mW).	9
PRF_i	- average repetition rate of impulsive interference (pulses/s).	
QPSK	- quadrature-phase-shift keyed.	144
$R_r\{\Delta f\}$	- insertion loss of receiver IF-amplifier filter (dB).	49
$R_t\{\Delta f\}$	- insertion loss of transmitter filter (dB).	54
SDR	- signal discontinuity rejection (dB).	99
S/I	- signal-to-interference power ratio (dB).	64

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$(S/I)_{cw}$	- signal-to-interference protective ratio for cw interference (dB).	66
$(S/\hat{I})_r$	- signal-to-peak interference protective ratio at output of IF amplifier (dB).	62
S/N	- signal-to-noise ratio (dB).	62
S/N_r	- signal-to-noise ratio at output of the IF amplifier (dB).	62
S_p	- spectrum peaking factor (dB).	55
SS	- spread spectrum.	xi
T_g	- average spacing of bursts in a gated SS signal (μs).	51
T_f	- average time for a frequency hopper to sequence through all of the available frequencies (μs).	52
$Z_t\{\Delta f\}$	- a function that is used to bridge the nulls in the spectral function $F_t\{\Delta f\}$ when calculating the OFR (dB).	96
$Z_\delta\{\Delta f\}$	- a function that is used to bridge the nulls in the spectral function $F_\delta\{\Delta f\}$ when calculating the OFR (dB).	92
δ	- effective rise or fall time of trapezoidal pulse or chip (from 0 to 100% amplitude) (μs).	50
τ	- duration of one chip (μs).	50
τ_g	- duration of one burst of a gated SS signal (μs).	51
τ_f	- dwell time on one frequency in a frequency-hopping sequence (μs).	52
τ_i	- duration of one bit of the information or baseband signal (μs).	52
$\oplus$	- modulo-2 addition.	140
Min $[x, y]$	- denotes the minimum of the two arguments.	16

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Max [x, y]	- denotes the maximum of the two arguments.	21
τ_s	- pulse length of signal generated by dispersive delay line or polychromatic delay line.	50

SECTION 1

INTRODUCTION

BACKGROUND

Spread-spectrum (SS) modulation techniques are capable of providing a high degree of protection against jamming as well as providing other desirable benefits in military telecommunications and radiolocation systems. The numerous development programs sponsored by the military have produced a wide variety of SS techniques and designs. Although extensive advancements have been made in the technology, there are still many unknowns in regard to the application of the techniques to military operations and the establishment of frequency allocation policies and regulations.

Some of the items that need to be resolved are: identification of the problems; determination of the compromises that have to be made in operational and frequency-allocation procedures; and determination of the possible parameter trade-offs. Resolving these items will entail performance of EMC (electromagnetic compatibility) analyses.

The matter of first priority is generally agreed to be the compatibility between SS transmitters and existing receiving systems. Obviously, this must be resolved before the telecommunications community will accept wide-scale use of SS systems.

This report presents analytical procedures for determining the electromagnetic compatibility of SS transmitters and conventional receiving systems. The procedures, which are generally applicable to all types of SS signals and to narrow- and wide-band receivers, involve simple formulas that yield accuracies that are in keeping with the requirements of most EMC analysis.

OBJECTIVES

The general objective was to provide an analytical approach for use in resolving the problems cited above. The specific objectives were to provide:

1. Analytical procedures for predicting the nature of the waveforms and the interference-to-noise ratios (INR's) associated with the responses to SS signals received by conventional radio systems, including wideband as well as narrowband receivers.
2. Guidelines for selecting appropriate INR threshold values for use as compatibility criteria.

APPROACH

The procedures presented here entail predicting the nature of the response waveform (noise-like, impulsive, CW-like, or undistorted) and calculating the INR of the response at the IF amplifier output. The INR is then compared to an INR threshold, which is selected on the basis of the nature of the response waveform and on the type of data processor used in the receiver. Guidelines that can be used for selecting the threshold are also provided.

In some EMC analyses, especially when the signal-to-noise ratio (S/N) is larger than the minimum required value, the signal-to-interference ratio (S/I) is calculated and compared with an S/I threshold to determine whether an unacceptable interference condition exists. If both the INR and the S/N in the IF bandwidth are known, it is a simple matter to obtain the S/I. Converting from an INR threshold to an S/I threshold, or vice versa, is usually also easy to do, as will be explained later.

Model For Receiving Channel

The actual receiving channel consists of the front end, IF amplifier, and the data processor, as depicted in the block diagram in Figure 1. In calculating the signal power at the receiver input (point 1 in the figure) the antenna gains and the radio-transmission loss are taken into account. The INR at the IF-amplifier output (point 2 in the figure) is determined using an equation that was derived on the assumption that the front end and IF amplifier can be represented by a bandpass filter of fourth order or higher that has a bandwidth equal to the IF-amplifier bandwidth. Although this procdure treats the receiver as a linear device, it is usually adequate, because the interference threshold is generally set within the linear region of operation.

(a) Block diagram of receiving channel.

(b) Mathematical model of front end and IF amplifier.

Figure 1. Receiver representation for analysis.

The secondary effects, such as those caused by the intermodulation of multiple signals, are not included in the INR equation.

Primary Sources of Information

The formulas for calculating the rejection terms in the INR equation were derived from the results of an ECAC model development project that involves the analysis of responses produced by digital signals.[2]

Technical publications pertaining to the generation of pseudorandom, binary sequences and the responses they produce in narrowband filters were very helpful in deriving the INR equation. R. Dixon's short course on SS techniques and his recently published book (Reference 1) were the major sources of information on the various categories of SS techniques.

Thresholds of INR and S/I that can be used as compatibility criteria in situations that involve various kinds of interference waveforms and types of receiving systems have been established in previous investigations, many of which have been performed at ECAC. The published results of these investigations were reviewed so that guidelines could be established for selecting appropriate threshold values to be used in conjunction with the INR equation.

[2]Newhouse, P. D., *Procedures For Approximating the Peak Power of the Response of a Bandpass Filter to Pulses*, ECAC-TN-78-008, ECAC, Annapolis, MD, June 1978.

SECTION 2

THE ANALYTICAL PROCEDURES

The procedures for predicting the type of response waveform, calculating the INR, and selecting an INR-threshold are outlined in this section. To aid the reader in using the procedures, explanations are given of the parameters that appear in the equations and formulas. More detailed explanations and some tutorial material are given in the appendixes.

PREDICTION OF THE KIND OF RESPONSE WAVEFORM

The four basic types of responses that can be produced by SS signals are illustrated in Figure 2. The bandwidth conditions that are used to predict which kind of response will occur are given in TABLES 2 through 8, which appear later. Plots of response waveforms for a variety of bandwidth conditions are presented in APPENDIX E.

The waveforms shown throughout this report are hypothetical in that they are the envelopes of the responses of bandpass filters to SS signals in the absence of any other signals. In practice, the offending signals are usually accompanied by receiver noise and the desired signals, which become perturbed. If the perturbations are severe enough, an unacceptable interference condition is produced. APPENDIX D briefly explains how these perturbations occur in AM, FM, and PM receivers.

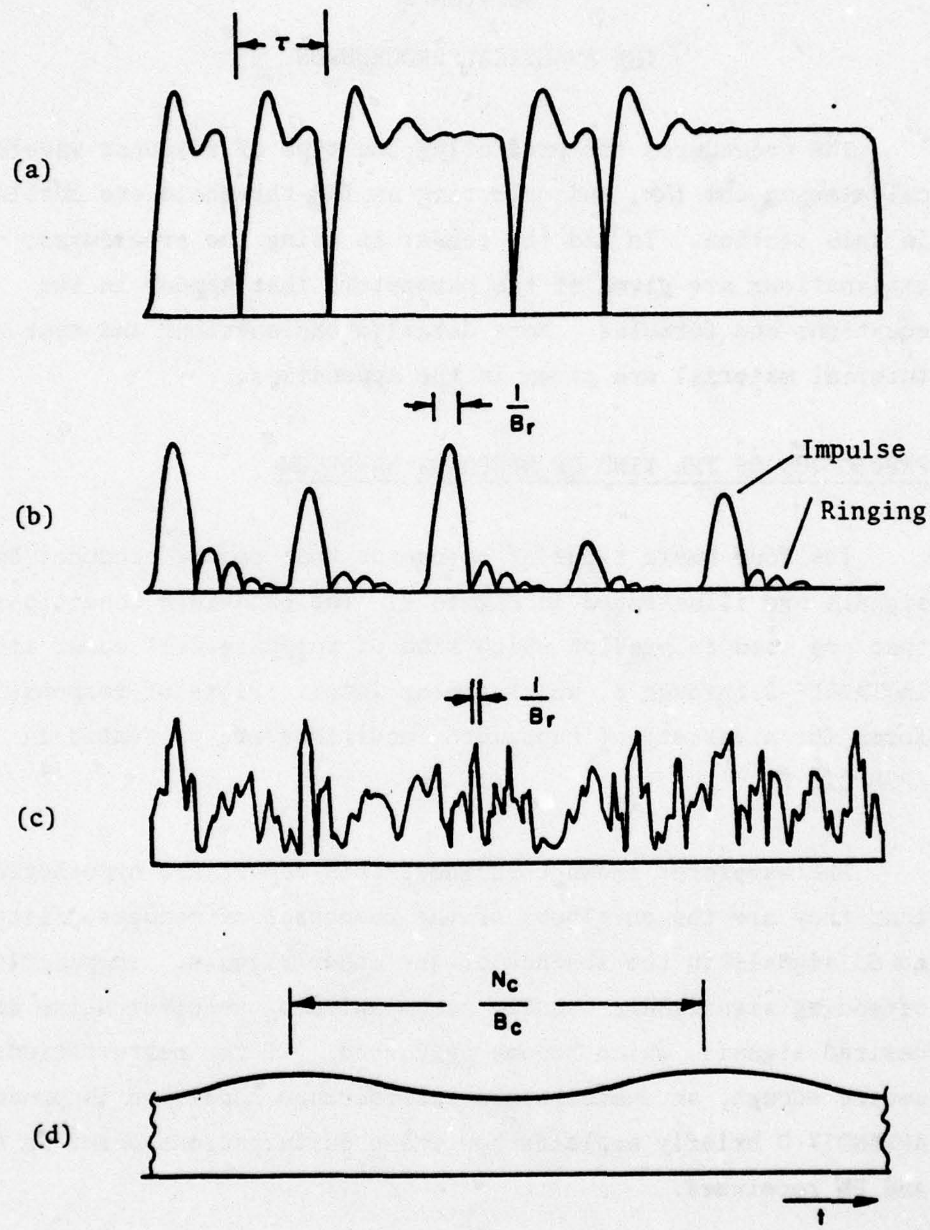

Figure 2. Types of response waveforms, (a) undistorted, (b) impulsive, (c) noise-like, (d) cw-like.

A prediction of the kind of response produced by an SS signal in the absence of other signals, even though such a condition is hypothetical, provides sufficient response-waveform information so that both a meaningful measure of interference power and an appropriate threshold can be selected.

Four categories of waveforms are used here, namely, the undistorted, impulsive, noise-like, and CW-like responses that were mentioned previously. These responses have the following characteristics:

Undistorted Response. This response is essentially the same as the input waveform except that it has been translated to the intermediate frequency. Each chip, which is of length τ, is distinguishable even though there may be some overshoot and rounding off, as illustrated in Figure 2(a). This kind of response may be most troublesome when the offending signal resembles the desired signal.

Impulsive Response. This response consists of impulses resulting from changes in the input signal. The impulses do not overlap one another and are therefore easily distinguishable. The peaks of the impulses may be of uniform amplitude or random, and their spacing may be either discrete or random depending on the conditions causing them. These conditions are delineated in TABLES 2 through 7, which appear later. The minimum impulse length is about $1/B_r$ as indicated in Figure 2(b). Two very important characteristics of an impulsive response are: the expected peak power and the average spacing of the impulses. Both of the characteristics can be predicted with the procedures given in the tables.

Noise-like Response. This response consists of impulses that overlap one another so that they are not distinguishable. The waveform is spiky or noisy, as illustrated in Figure 2(c). The minimum spike width is about $1/B_r$ as indicated. A typical crest factor (ratio of peak-to-average power) is 8 dB, for the noise-like responses in the calculated waveforms shown in APPENDIX E. In contrast, the crest factors for waveforms obtained with laboratory noise generators were found to be 14 dB. This indicates that a noise-like response produced by an SS signal is not as nearly Rayleigh as the response to a laboratory noise generator.

CW-like Response. This response has a bandwidth of less than B_r. Frequency components equal to B_c/N_c and its harmonics can be distinguished in the waveform.

* * *

Under some conditions, a response may appear to be a mixture of two of the above categories, or fall midway between two of them, such as the example shown in APPENDIX E, Figure E-8. This occurs when the bandwidth condition is near one of the limits. For example, with some signals, an impulsive response is predicted when $B_rT_g < 1$, and a noise-like response is predicted when $B_rT_g > 1$; when $B_rT_g \approx 1$, the response is not clearly impulsive or noise-like. When that occurs, the analyst can select the type of response that imposes the more severe INR threshold.

THE INR EQUATION

Two different measures of power are appropriate for the four types of responses. For undistorted, noise-like, and CW-like responses, the measure of power is the mean-square of the entire response waveform. For impulsive responses, the appropriate measure of power is the mean-square of only the peaks of the impulses and not of the entire waveform, which includes ringing that follows the impulses, as shown in Figure 2(b).

The INR for any of the four types of responses is obtained by:

$$INR = INR_i - OTR - OFR - SDR \qquad (1)$$

where INR_i is the interference-to-noise ratio at the terminals of the receiving antenna, that is,

$$INR_i = P_t + G_t + G_r - L_p - N_r \qquad (2)$$

and

P_t = SS transmitter power (dBm)

G_t = gain of SS transmitting antenna in direction of victim receiver (dBi)

G_r = gain of receiving antenna in direction of SS transmitter (dBi)

L_p = radio transmission loss between antennas (dB)

N_r = noise power in the receiver IF bandwidth (dBm)
= $-114 + F_r + 10 \log B_r$

B_r = IF-amplifier 3-dB bandwidth (MHz)

F_r = noise figure[3] of the receiving system, including ambient as well as thermal noise (dB)

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[3]*Reference Data for Radio Engineers,* International Telephone and Telegraph Corp., 1975, p. 29-2.

OTR = on-tune rejection (dB)

OFR = off-tune rejection (dB)

SDR = signal discontinuity rejection (dB)

The rejection factors, OTR, OFR, and SDR, are calculated with formulas that are given in TABLES 2, 4, 6, and 8, which appear later. APPENDIX B gives more detailed definitions and explanations of these factors.

Obtaining INR_i with Equation 2 entails calculating the radio-transmission loss (L_p) between the antennas of the offending SS transmitter and victim receiver. Procedures for calculation L_p are available in the literature and are not given in this report.

<u>Calculation of the Rejection Factors: OTR, OFR, SDR</u>

Three optional procedures are provided; they differ in the way in which OFR is calculated. By using the appropriate option, the amount of effort required to perform the calculations is minimized.

The Distinguishing Features of the Optional Procedures. Distinguishing features of the optional procedures are listed below in the order of increasing complexity.

Option No. 1 (Use TABLE 2)

(a) Applicable to cochannel situations when $B_t/B_r > 1$; is not applicable to adjacent-channel situations.

(b) OTR and SDR are calculated using an appropriate formula from TABLE 2.

(c) OFR is not required.

Option No. 2 (Use TABLES 3 through 8)

(a) Applicable to cochannel and adjacent-channel situations for any B_t/B_r.

(b) OTR, OFR, and SDR are calculated using appropriate formulas from TABLE 4, 6, or 8.

(c) Formula for OFR uses the spectral bounds, $\tilde{E}_t\{\Delta f\}$ of the transmitter emission spectrum. The bounds can be either calculated, using procedures given, or obtained from measurements made with a spectrum analyzer.

Option No. 3 (Use APPENDIX B and F)

Same as Option No. 2, except a spectral density function, $E_t\{\Delta f\}$ or $E_\delta\{\Delta f\}$ is used in place of the bounds in step (c). The mathematical expression for the spectral functions are obtained with Equation F-8 through F-29 in APPENDIX F. (See also APPENDIX B on use of the functions.)

If the SS signal is limited in the power amplifier of the SS transmitter, the spectrum may be altered. In this case, care should be exercised if a calculated spectral function is used in the analysis.

Guidelines for selecting the appropriate option are listed in TABLE 1.

TABLE 1

GUIDELINES FOR SELECTING THE APPROPRIATE OPTIONAL PROCEDURE FOR CALCULATING OTR, OFR, AND SDR

Option No.	Relative Simplicity	Applicable Conditions	Treatment Of OFR	Relative Accuracy	Appropriate Applications
1	Easiest to use	Cochannel with $B_t/B_r > 1$	Involves no OFR	Good ($\epsilon \pm 1$ dB)[a]	Preferred when $\Delta f \approx 0$ and $B_t/B_r > 1$
2	Intermediate	Cochannel or adjacent channel, with any B_t/B_r	Spectrum bounds used to calculate OFR	Good except when receiver is near nulls in spectrum	Preferred over Option No. 3 when $\Delta f \ll 1/2\delta$ & $B_c < 10$ Mc/s
3	Entails the greatest number of steps	Cochannel or adjacent channel, with any B_t/B_r	Spectrum used to calculate OFR	Comparable to Option No. 1	Preferred when $B_c > 10$ Mc/s or $\Delta f > 1/(4\delta)$

[a] ϵ = theoretical error in calculated rejection factors.

Comparison of Options 2 and 3

The difference in the results obtained with Options 2 and 3 is illustrated graphically in Figure B-7 in APPENDIX B, which shows a plot of the OFR obtained with the two methods. The example shown in Figure B-7 is representative of situations in which $B_t/B_r \ll 1$.

The difference in the results with the two options is greatest when the receiver is tuned to a null in the signal spectrum, with Option No. 2 giving the more conservative prediction (i.e., predicting less OFR than actually occurs).

Manual Calculations vs Computer

The formulas used in Options No. 1 and 2 are simple enough so that they can be evaluated easily using manual computations. The formulas in Option No. 3 are only slightly more difficult for someone who is acquainted with the procedures. If the procedures are to be automated (programmable calculator or computer), Option No. 3 should be included because of the greater accuracy it provides in representing the emmission spectra, which is important (especially in some frequency-hopping applications).

Accuracy

The formulas given here for calculating OTR and OFR yield results suitable for most EMC analyses. That is, the computational errors are typically less than the variances of the input data. In the case of PSK signals with no transmitter filter, the formulas used in Option No. 3 were found to be generally accurate to within 2 dB or less, as illustrated in APPENDIX B.

GUIDELINES FOR USING PROCEDURES GIVEN IN THIS SECTION

1. Determine the type of response. Use TABLE 2, 3, 5, or 7.

(a) Select table. Use TABLE 2 if cochannel operation is involved and $1 < B_t/B_r < N_c$; otherwise, use TABLE 3 for direct-sequence or pulsed-FM signals; TABLE 5 for frequency-hopping signals; or TABLE 7 for hybrid signals.

(b) Selecte the appropriate line on the table. Selection is made by finding which set of signal/bandwidth conditions fits the parameters of the equipment being analyzed.

(c) Having selected the appropriate line on the table, refer to column headed "Type of Response."

2. Determine OTR, OFR, and SDR. Use the formulas given in the appropriate table or subtable.

In TABLE 2, the formulas are given for OTR and SDR. In the other tables, the column headed "Formulas" gives the subtable number that gives the formulas for OTR, OFR, and SDR.

The various parameters used in the formulas are explained in Section 2 (under the subheading "Explanation of Parameters Used").

Additional guidelines for plotting OFR as a function of Δf are given on page 15.

3. Having calculated OTR, OFR, and SDR, calculate the INR using Equations 1 and 2.

4. Choose the value to be used for the interference-to-noise- ratio threshold (INR_t) using the procedure explained under the heading "INR THRESHOLD."

5. Compare calculated value of INR with INR_t. The equipments satisfy the criterion for compatibility if $INR < INR_t$.

6. Refer to the subsection "APPLICATION OF INR EQUATION" for additional guidance, especially when a frequency-hopping signal is involved.

GUIDELINES FOR PLOTTING OFR

Plotting OFR as a function of $|\Delta f|$ generally entails the steps described below:

For Narrowband Receiver, i.e., $c_1 < B_t/B_r < N_c$.

(a) Plot $\tilde{F}_t\{\Delta f\}$ (Figure 3(a)) using the procedure given in Figure 10, 11, or 12, depending on the type of modulation.
(b) Plot $R_t\{\Delta f\}$, (Figure 3 (b)), using Equation 4.
(c) Plot the bounds on the emission spectrum, $\tilde{E}_t\{\Delta f\} = -R_t\{\Delta f\} + \tilde{F}_t\{\Delta f\}$, (Figure 3(c)). Note that $\tilde{E}_t\{\Delta f\}$ denotes the bounds on the filtered signal spectrum. If a spectrum analyzer photograph of the emission spectrum is available, the bounds on that can be used in place of $\tilde{E}_t\{\Delta f\}$. For a narrowband receiver OFR = $-\tilde{E}_t\{\Delta f\}$ as indicated in Figure 3(d).

For a Wide-band Receiver, i.e., $B_t/B_r < c$.

(a) Plot $\tilde{F}_t\{\Delta f\}$, as step (a) above.
(b) Plot $R_t\{\Delta f\}$, as step (b) above.
(c) Plot $\tilde{E}_t\{\Delta f\}$, as step (c) above.
(d) Calculate C_{wb}, using appropriate formula from TABLE 9, and plot $[\tilde{E}_t\{\Delta f\} + C_{wb}]$, (Figure 4(a)).
(e) Plot $R_r\{\Delta f\}$, (Figure 4(b)), using Equation 3.

(f) Plot OFR. An approximation can be obtained by

$$\mathrm{OFR} = \begin{cases} 0, \text{ for } |\Delta f| < B_r/2 \\ \mathrm{Min}\left[R_r\{\Delta f\},\ (-\tilde{E}_t\{\Delta f\} - C_{wb})\right], \text{ otherwise.} \end{cases}$$

Figure 4(c) shows the various segments of $R_r\{\Delta f\}$ and $(-\tilde{E}_t\{\Delta f\} - C_{wb})$ that are used to describe OFR. For clarity, OFR is also shown in Figure 4(d).

A more accurate approximation is $\mathrm{OFR} = \mathrm{Min}\left[0,\ R_I\{\Delta f\}\right]$ where $R_I\{\Delta f\}$ is obtained using the formula given in TABLE 4, 6, or 8. A comparison of the two methods is shown in Figure 5. Note that the two methods yield results that differ by 6 dB at $|\Delta f|$ where $R_r\{\Delta f\} = \left[(-\tilde{E}_t\{\Delta f\} - C_{wb})\right]$, but for much smaller or larger values of $|\Delta f|$ the difference tends to 0 dB.

Sample calculations that illustrate the application of the procedures are presented in APPENDIX G.

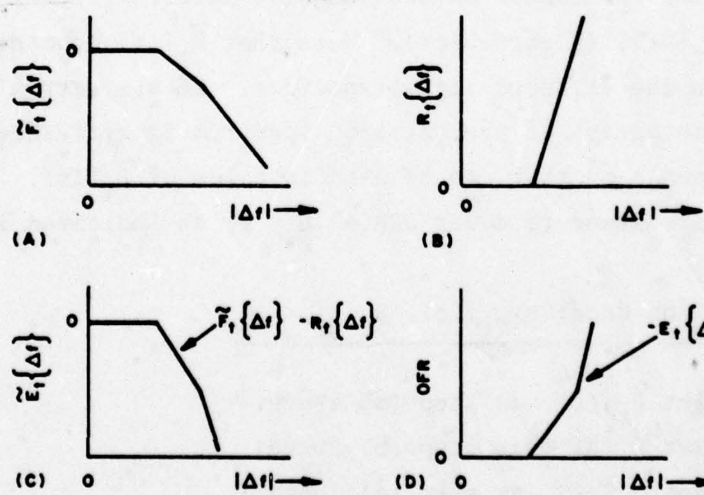


Figure 3. Plots used in calculating and plotting OFR for narrowband receivers. (ordinates are in dB and abscissas in log $|\Delta f|$.

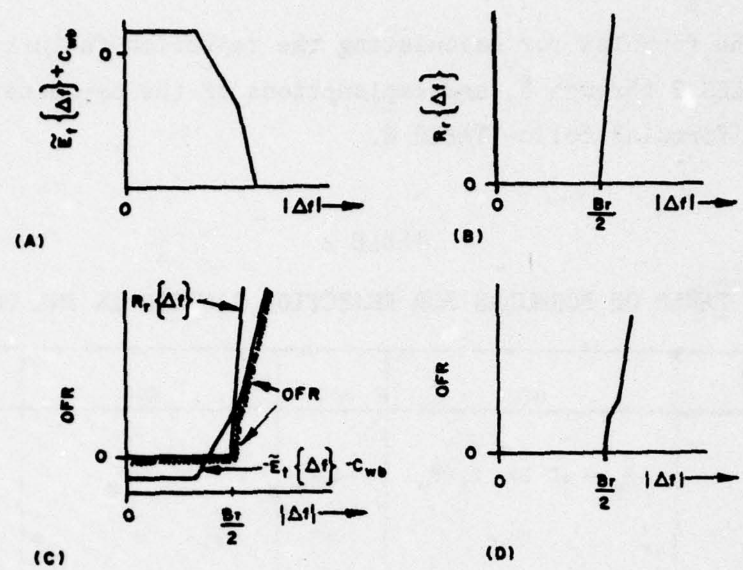


Figure 4. Plots used in calculating and plotting OFR for wideband receivers (ordinates in dB and abscissas in log $|\Delta f|$).

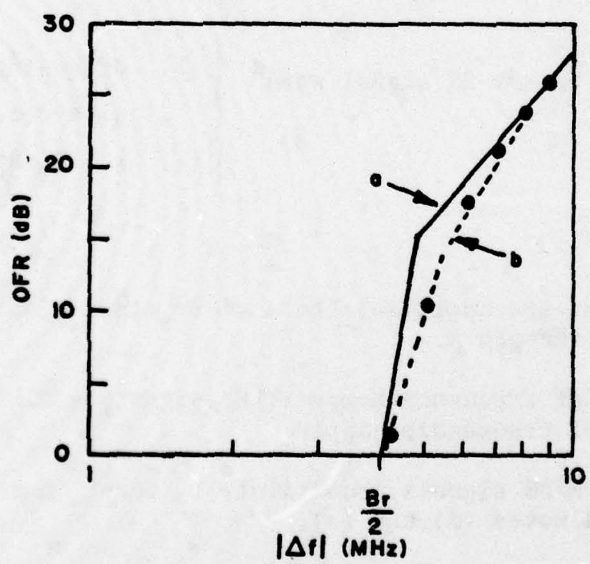


Figure 5. A comparison of OFR calculated using

(a) OFR = Min $[R_r\{\Delta f\}, (-\tilde{E}_t\{\Delta f\} - C_{wb})]$, for $|\Delta f| > B_r/2$

(b) OFR = Min $[0, R_I\{\Delta f\}]$

The formulas for calculating the rejection factors are given in TABLES 2 through 8, and explanations of the parameters appearing in the formulas follow TABLE 8.

TABLE 2

SHORT TABLE OF FORMULAS FOR REJECTION FACTORS IN INR EQUATION

Alternative Conditions[c]	OTR	OFR	SDR	Type of Response
Signal not gated	$-S_p + 10 \log B_t/B_r$	0	0	Noise-like
$1/B_r < \tau_g$	"	0	0	Noise-like bursts[d]
$\tau_g < 1/B_r < T_g$	"	0	$10 \log (B_r\tau_g)^{-1}$	Impulsive[e]
$T_g < 1/B_r$	"	0	$10 \log (T_g/\tau_g)$	Noise-like

Applicable to any SS signal when[a] (1) offending signal is cochannel with receiver,[b] and (2) $1 < B_t/B_r < N_c$

Notes

[a]Formulas that are applicable to a wider range of conditions are given in TABLES 3 through 8.

[b]In the case of frequency-hopped (FH) signals and hybrids (FH/DS), only the cochannel frequencies apply.

[c]For FH and FH/DS signals, substitute τ_f for τ_g and T_f for T_g in the formulas and notes (d) and (e).

[d]The burst length is $\approx \tau_g$ and the average spacing is T_g.

[e]The average spacing between impulses is T_g.

TABLE 3

FORMULAS FOR REJECTION FACTORS
USED IN THE INR EQUATION
(DIRECT SEQUENCY OR PULSED-FM SIGNALS)

Signal/Bandwidth Conditions [a]							
SIGNAL CONTINUITY	B_t/B_r	$\lvert\Delta f\rvert$	$B_r\tau_g$	B_rT_g	B_t/B_i	Type of Response Waveform	Formulas (see following tables)
Uninterrupted [b]	* < c_1	* ≤ $B_r/2$				Undistorted	4a
"	"	* > $B_r/2$				Impulsive or undistorted	4b
"	c_1 < * < N_c	* ≥ 0				Noise-like	4c
"	* > N_c	"			* ≤ N_c	Noise-like	4d
"	"	"			* > N_c	cw-like	4e
Gated [c]	* < c_1	* ≤ $B_r/2$				Undistorted	4f
"	"	* > $B_r/2$				Impulsive	4g
"	c_1 < * < N_c	* ≥ 0	* > 1			Noise-like	4h
"	"	"	* < 1	* > 1		Impulsive or undistorted	4i
"	"	"	"	* < 1		Noise-like	4j
"	* > N_c	"	"	"	* ≤ N_c	Noise-like	4k
"	"	"	"	"	* > N_c	cw-like	4l

[a]For each of these entries, substitute the column heading (B_t/B_r), $|\Delta f|$, etc) in place of the star (*) printed in the colums.

[b]The transmission is considered to be uninterrupted when the duration is greater than several seconds.

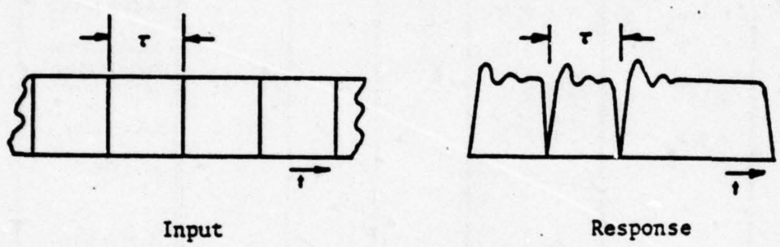
[c]The gate, or burst, length is τ_g and the spacing T_g.

TABLE 4-a

RESPONSE CHARACTERISTICS AND FORMULAS
(DIRECT SEQUENCE OR PULSED-FM SIGNALS)

Signal/bandwidth condition:

$(B_t/B_r < c_1)$ and $(|\Delta f| \leq B_r/2)$



Envelopes of Waveforms

Signal continuity: Uninterrupted

Type of response: Undistorted

Crest factor: ≈0 dB

Formulas for rejection factors:

OTR = 0

OFR = 0

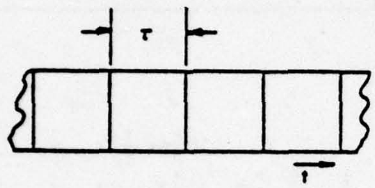
SDR = 0

Suggested INR threshold: $INR_t = (S/N)_r - (S/I_{ud})$
(See Equation 20)

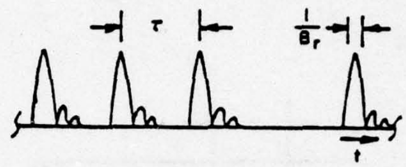
TABLE 4-b

RESPONSE CHARACTERISTICS AND FORMULAS
(DIRECT SEQUENCE OR PULSED-FM SIGNALS)

Signal/bandwidth condition:
$(B_t/B_r < c_1)$ and $(|\Delta f| > B_r/2)$



Input



Response [c]

Envelopes of Waveforms

Signal continuity: Uninterrupted

Type of response: Impulsive[(a)] or undistorted[c]

Crest factor: Not applicable

Formulas for rejection factors:[(b)]

$$OTR = 0$$

$$OFR = \mathrm{Max}[0,\ R_I\{\Delta f\}]$$

$$R_I\{\Delta f\} = -20 \log \left\{ \mathrm{antilog}\left[\frac{E_t\{\Delta f\} + C_{Wb}}{20}\right] + \mathrm{antilog}\left[\frac{-R_r\{\Delta f\}}{20}\right] \right\}$$

$$\tilde{E}_t\{\Delta f\} = F_t\{\Delta f\} = R_t\{\Delta f\}$$

$$SDR = 0$$

Suggested INR threshold: $INR_t = (S/N)_r - (S/\hat{I})_r - PF$ See Equation 16

Notes: [a]Impulses have discrete amplitudes and pseudorandom spacing.

[b]When used in the INR equation, these factors yield the INR of the peaks of the impulses.

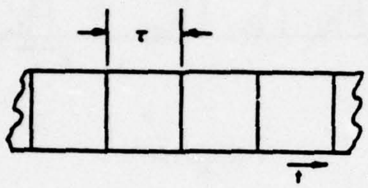
[c]When $R_r\{\Delta f\} < -(C_{wb} + E_t\{\Delta f\})$ the response waveform is essentially undistorted and the INR threshold suggested in TABLE 4a should be used.

TABLE 4-c

RESPONSE CHARACTERISTICS AND FORMULAS
(DIRECT SEQUENCE OR PULSED-FM SIGNALS)

Signal/bandwidth condition:

$(c_1 < B_t/B_r < N_c)$ and $(|\Delta f| \geq 0)$



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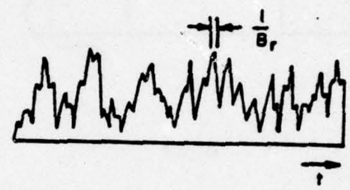


Response

Envelopes of Waveforms

Signal continuity: Uninterrupted

Type of response: Noise-like

Crest factor: ≈8 dB

Formulas for rejection factors:

$$OTR = -S_p + 10 \log (B_t/B_r)$$

$$OFR = -\widetilde{E}_t\{\Delta f\}$$

$$SDR = 0$$

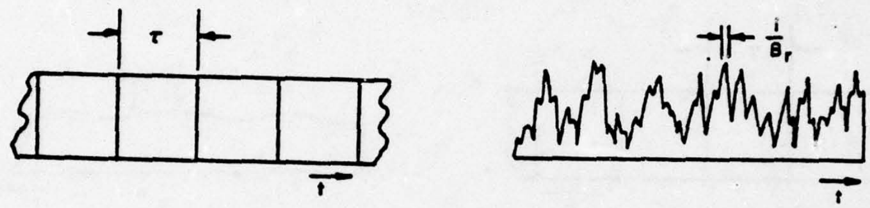
Suggested INR threshold: $INR_t = 10 \log (-1 + 10^{C/10})$

TABLE 4-d

RESPONSE CHARACTERISTICS AND FORMULAS
(DIRECT SEQUENCE OR PULSED-FM SIGNALS)

Signal/bandwidth condition:

$(B_t/B_r > N_c)$, $(|\Delta f| \geq 0)$, and $(B_t/B_i < N_c)$



Input Response

Envelopes of Waveforms

Signal continuity: Uninterrupted

Type of response: Noise-like

Crest factor: ≈8 dB

Formulas for rejection factors:

$$OTR = -S_p + 10 \log (B_t/B_r)$$

$$OFR = -\tilde{E}_t\{\Delta f\}$$

$$SDR = 0$$

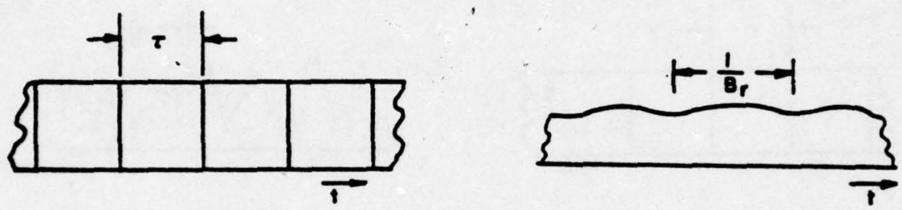
Suggested INR threshold: $INR_t = 10 \log (-1 + 10^{C/10})$

TABLE 4-e

RESPONSE CHARACTERISTICS AND FORMULAS
(DIRECT SEQUENCE OR PULSED-FM SIGNALS)

Signal/bandwidth condition:

$(B_t/B_r > N_c)$, $(|\Delta f| > 0)$, and $(B_t/B_i > N_c)$



Input **Response**

Envelopes of Waveforms

Signal continuity: Uninterrupted

Type of response: cw-like

Crest factor: ≈3 dB

Formulas for rejection factors:

$$OTR \geq -S_p + 10 \log N_c \qquad \text{(See Note a)}$$

$$OFR = -\tilde{E}_t\{\Delta f\}$$

$$SDR = 0$$

Suggested INR threshold: $INR_t = (S/N)_r - (S/I_{cw})$ See Equation 19.

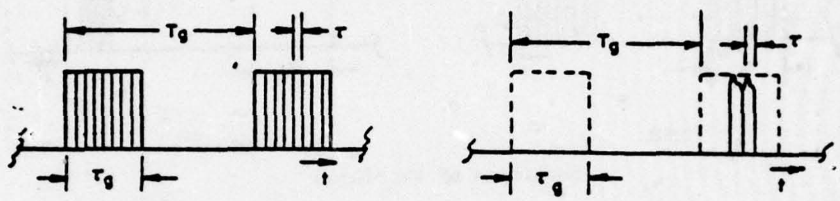
Note: [a] The center line of the spectrum of the SS signal is suppressed, therefore the OTR is highly dependent on the rejection of the adjacent lines. See Appendix B.

TABLE 4-f

RESPONSE CHARACTERISTICS AND FORMULAS (DIRECT SEQUENCE OR PULSED-FM SIGNALS)

Signal/bandwidth condition:

$(B_t/B_r < c_1)$ and $(|\Delta f| \leq B_r/2)$



Input **Response**

Envelopes of Waveforms

Signal continuity: Gated

Type of response: Undistorted

Crest factor: ≈ 0 dB

Formulas for rejection factors: [a]

OTR = 0

OFR = 0

SDR = 0

Suggested INR threshold: $INR_t = (S/N)_r - (S/I_{ud})$ See Equation 20.

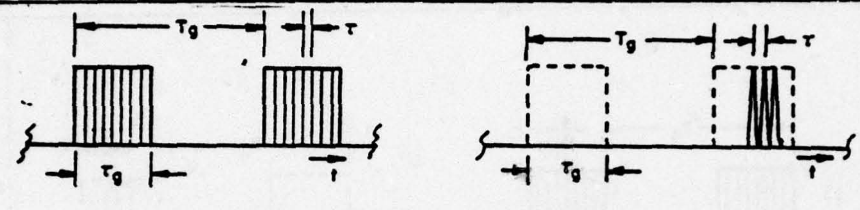
Note: [a] When used in the INR equations, these factors yield the INR (mean-square of the amplitude) of the response during the gate length τ_g.

TABLE 4-g

RESPONSE CHARACTERISTICS AND FORMULAS
(DIRECT SEQUENCE OR PULSED-FM SIGNALS)

Signal/bandwidth condition:

$(B_t/B_r < c_1)$ and $(|\Delta f| > B_r/2)$



Envelopes of Waveforms

Signal continuity: Gated

Type of response: Impulsive[a] uniform amplitude, or undistorted[c]

Crest factor: Not applicable

OTR = 0

OFR = Max $[0, R_I\{\Delta f\}]$ (See Note b)

$$R_I\{\Delta f\} = -20 \log \left\{ \text{antilog} \left[\frac{\tilde{E}_t\{\Delta f\} + C_{wb}}{20} \right] + \text{antilog} \left[\frac{-R_r\{\Delta f\}}{20} \right] \right\}$$

$$\tilde{E}_t\{\Delta f\} = \tilde{F}_t\{\Delta f\} - R_t\{\Delta f\}$$

SDR = 0

Suggested INR Threshold: $INR_t = (S/N)_r - (S/\hat{I})_r - PF$ (See Equation 16)

Notes: [a]Except for the leading and trailing impulses of each burst, the impulses have discrete amplitudes and pseudorandom spacing.

[b]When used in the INR equation, these factors yield the INR of the peaks of the impulses.

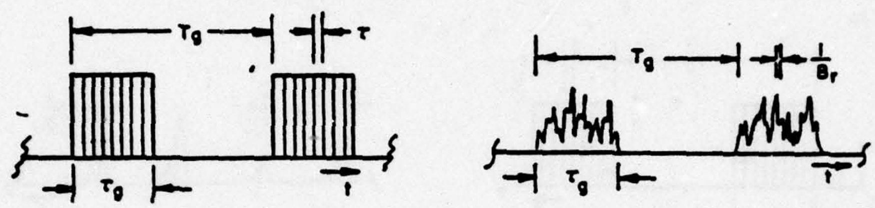
[c]When $R_r\{\Delta f\} < -(C_{wb} + \tilde{F}_t\{\Delta f\})$, the response waveform is essentially undistorted, as in TABLE 4a.

TABLE 4-h

RESPONSE CHARACTERISTICS AND FORMULAS
(DIRECT SEQUENCE OR PULSED-FM SIGNALS)

Signal/bandwidth condition:

$(c_1 < B_t/B_r < N_c)$, $(|\Delta f| \geq 0)$, and $(B_r\tau_g > 1)$



Input **Response**

Envelopes of Waveforms

Signal continuity: Gated

Type of response: Noise-like

Crest factor: ≈8 dB

Formulas for rejection factors: [a]

$$OTR = -S_p + 10 \log (B_t/B_r)$$

$$OFR = -\tilde{E}_t\{\Delta f\}$$

$$SDR = 0$$

Suggested INR threshold: $INR_t = 10 \log (-1 + 10^{C/10})$

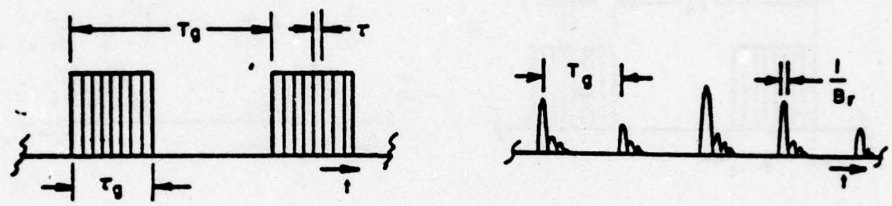
Note: [a]When used in the INR equation, these factors yield the INR (mean-square of the amplitude) of the response during the gate length τ_g.

TABLE 4-i

RESPONSE CHARACTERISTICS AND FORMULAS
(DIRECT SEQUENCE OR PULSED-FM SIGNALS)

Signal/bandwidth condition:

$(c_1 < B_t/B_r < N_c)$, $(|\Delta f| \geq 0)$, $(B_r\tau_g < 1)$, and $(B_rT_g > 1)$



Input Response

Envelopes of Waveforms

Signal continuity: Gated

Type of response: Impulsive [a]

Crest factor: Not applicable (See note b)

Formulas for rejection factors: [b]

$$OTR = -S_p + 10 \log (B_t/B_r)$$

$$OFR = -\tilde{E}_t\{\Delta f\}$$

$$SDR = 10 \log (B_r\tau_g)^{-1}$$

Suggested INR threshold: $INR_t = (S/N)_r - (S/\hat{I})_r - PF$ (See Equation 16)

Notes: [a] The impulses have pseudorandom amplitudes and uniform spacing.

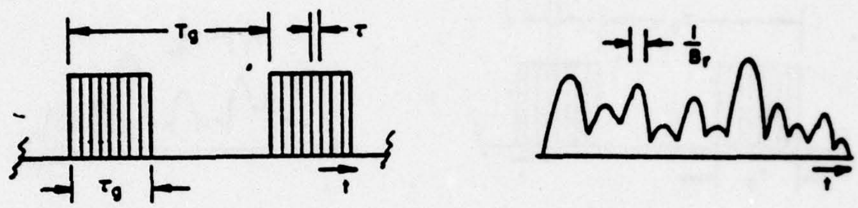
[b] Using these factors in the INR equation yields the expected INR of the peaks of the impulses. The maximum peak value is about 8 dB above the expected value.

TABLE 4-j

RESPONSE CHARACTERISTICS AND FORMULAS
(DIRECT SEQUENCE OR PULSED-FM SIGNALS)

Signal/bandwidth condition:

$(c_1 < B_t/B_r < N_c)$, $(|\Delta f| \geq 0)$, $(B_r\tau_g < 1)$, and $(B_rT_g < 1)$



Input **Response**

Envelopes of Waveforms

Signal continuity: Gated

Type of response: Noise-like

Crest factor: ≈8 dB

Formulas for rejection factors:

$$\text{OTR} = -S_p + 10 \log (B_t/B_r)$$

$$\text{OFR} = -\tilde{E}_t\{\Delta f\}$$

$$\text{SDR} = 10 \log (T_g/\tau_g)$$

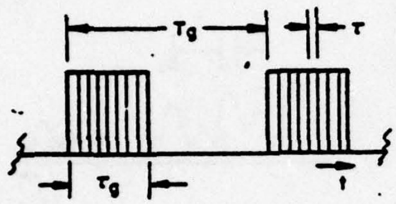
Suggested INR threshold: $\text{INR}_t = 10 \log (-1 + 10^{C/10})$

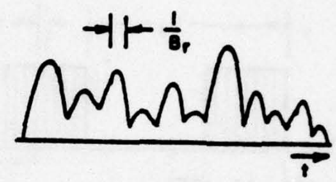
TABLE 4-k

RESPONSE CHARACTERISTICS AND FORMULAS
(DIRECT SEQUENCE OR PULSED-FM SIGNALS)

Signal/bandwidth condition:

$(B_t/B_r > N_c)$, $(|\Delta f| \geq 0)$, $(B_r \tau_g < 1)$, $(B_r T_g < 1)$, and $(B_t/B_i \leq N_c)$





Input **Response**

Envelopes of Waveforms

Signal continuity: Gated

Type of response: Noise-like

Crest factor: ≈8 dB

Formulas for rejection factors:

$$\text{OTR} \gtrsim -S_p + 10 \log B_t/B_r$$

$$\text{OFR} = -\tilde{E}_t\{\Delta t\}$$

$$\text{SDR} = 10 \log (T_g/\tau_g)$$

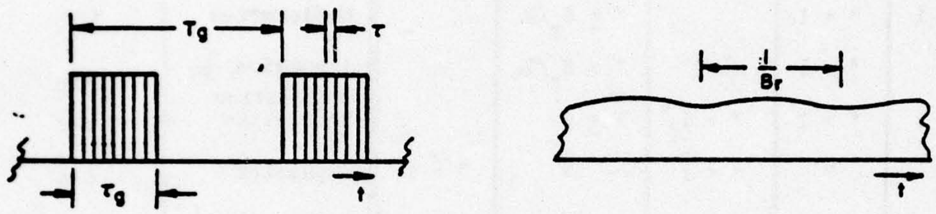
Suggested INR threshold: $\text{INR}_t = 10 \log (-1 + 10^{C/10})$

TABLE 4-1

RESPONSE CHARACTERISTICS AND FORMULAS (DIRECT SEQUENCE OR PULSED-FM SIGNALS)

Signal/bandwidth condition:

$(B_t/B_r > N_c)$, $(|\Delta f| \geq 0)$, $(B_r\tau_g < 1)$, $(B_rT_g < 1)$, $(B_t/B_i > N_c)$



Input **Response**

Envelopes of Waveforms

Signal continuity: Gated

Type of response: cw-like

Crest factor: ≈3 dB

Formulas for rejection factors:

OTR $\geqq -S_p + 10 \log N_c$ (See Note a)

OFR $= -\tilde{E}_t\{\Delta f\}$

SDR $= 10 \log (T_g/\tau_g)$

Suggested INR threshold: $INR_t = (S/N)_r - (S/I_{cw})$. See Equation 19

Note: [a]The center line of the spectrum of the SS signal is suppressed, therefore the OTR is highly dependent on the rejection of the adjacent lines. See Appendix B.

TABLE 5

FORMULAS FOR REJECTION FACTORS
USED IN INR EQUATION
(FREQUENCY-HOPPED SIGNALS)

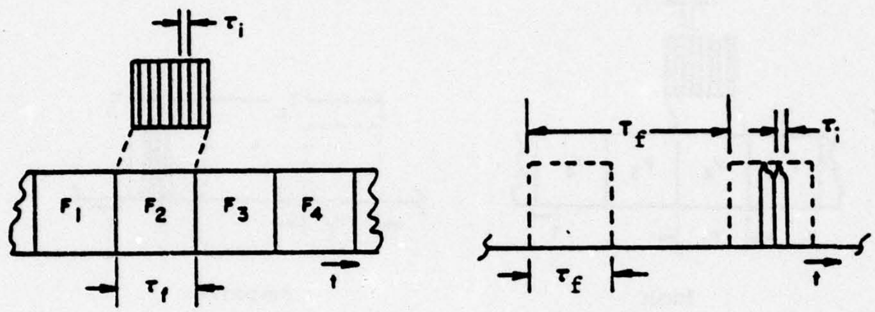
Signal/Bandwidth Conditions[a]					Type of Response Waveform	Formulas (see following tables)
$B_i\tau_f$	B_i/B_r	$B_r\tau_f$	$\lvert\Delta f_j\rvert$	B_rT_f		
* > 1	* < 1		* $\leq B_r/2$		Undistorted	6-a
"	* < 1		* $> B_r/2$		Impulsive, or Undistorted	6-b
"	* > 1	* > 1	* ≥ 0		Noise-like	6-c
"	"	* < 1	"	* > 1	Impulsive	6-d
"	"	"	"	* < 1	Noise-like	6-e
* < 1		* > 1	* $\leq B_r/2$		Undistorted	6-f
"		"	* $> B_r/2$		Impulsive, or Undistorted	6-g
"		* < 1	* ≥ 0	* > 1	Impulsive	6-h
"		"	* ≥ 0	* < 1	Noise-like	6-i

[a]For each of these entries, substitute the column heading ($B_i\tau_f$, B_i/B_r, etc.) in place of the asterick (*) printed in the columns.

TABLE 6-a

RESPONSE CHARACTERISTICS AND FORMULAS
(FREQUENCY-HOPPED SIGNALS)

Signal/bandwidth condition:
$(B_i \tau_f > 1)$, $(B_i/B_r < 1)$, and $(|\Delta f_j| \leq B_r/2)$



Input Response

Envelopes of Waveforms

Signal continuity: Dwell time on a frequency = τ_f

Type of response: Undistorted

Crest factor: ≈0 dB

Formulas for rejection factors:[a]

OTR = 0

OFR = 0

SDR = 0

Suggested INR threshold: $INR_t = (S/N)_r - (S/I_{ud})$ See Equation 20

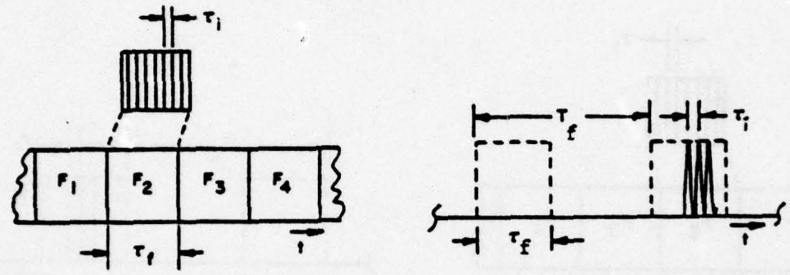
Note: [a] Using these factors in the INR equation yields the INR of the response during the dwell time τ_f produced by the jth frequency of the SS signal.

TABLE 6-b

RESPONSE CHARACTERISTICS AND FORMULAS (FREQUENCY-HOPPED SIGNALS)

Signal/bandwidth condition:

$(B_i\tau_f > 1)$, $(B_i/B_r < 1)$, and $(|\Delta f_j| > B_r/2)$



Input Response[c]

Envelopes of Waveforms

Signal continuity: Dwell time on one frequency = τ_f

Type of response: Impulsive,[a] or indistorted[c]

Crest factor: Not applicable

Formulas for rejection factors:[b]

$$OTR = 0$$

$$OTR_j = \text{Max}\,[0,\ R_I\{\Delta f_j\}]$$

$$R_I\{\Delta f_j\} = -20\log\left\{\text{antilog}\left[\frac{\tilde{E}_i\{\Delta f_j\} + C_{Wb}}{20}\right] + \text{antilog}\left[\frac{-R_r\{\Delta f_j\}}{20}\right]\right\}$$

$$\tilde{E}_i\{\Delta f_j\} = \tilde{F}_i\{\Delta f_j\} - R_t\{\Delta f_o\}$$

$$SDR = 0$$

Suggested INR threshold: $INR_t = (S/N)_r - (S/\hat{I})_r - PF$ (See Equation 16)

Notes:

[a]Except for the leading and trailing impulses of each burst, the impulses have uniform amplitude and random spacing.

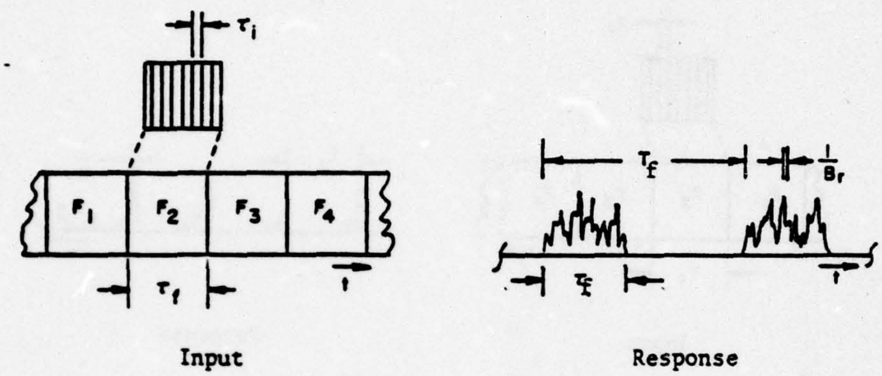
[b]When used in the INR equation, these factors yield the INR of the peaks of the impulses produced by the jth frequency of the SS signal.

[c]When $R_r\{\Delta fj\} < -(C_{wb} + \tilde{F}_i\{\Delta f_o\})$, the response waveform is essentially undistorted as in TABLE 5a.

TABLE 6-c

RESPONSE CHARACTERISTICS AND FORMULAS
(FREQUENCY-HOPPED SIGNALS)

Signal/bandwidth condition:
$(B_i \tau_f > 1)$, $(B_i/B_r > 1)$, $(B_r \tau_f > 1)$, $(|\Delta f_j| \geq 0)$



Envelopes of Waveforms

Signal continuity: Dwell time on one frequency = τ_f

Type of response: Noise-like [a]

Crest factor: ≈8 dB

Formulas for rejection factors: [b]

OTR = $10 \log (B_i/B_r)$

OFR = $-\tilde{E}_i\{\Delta f_j\}$

SDR = 0

Suggested INR threshold: $INR_t = 10 \log (-1 + 10^{C/10})$

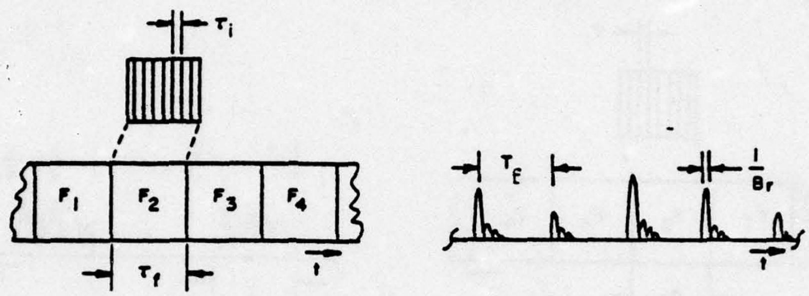
Notes: [a]This assumes that the information signal consists of a random sequence of bits.

[b]When used in the INR equations, these factors yield the INR of the response during the dwell time τ_f produced by the jth frequency of the SS signal.

TABLE 6-d

RESPONSE CHARACTERISTICS AND FORMULAS
(FREQUENCY-HOPPED SIGNALS)

Signal/bandwidth condition:
$(B_i\tau_f > 1)$, $(B_i/B_r > 1)$, $(B_r\tau_f < 1)$, $(B_rT_f > 1)$, and $(|\Delta f_j| \geq 0)$



Input Response

Envelopes of Waveforms

Signal continuity: Dwell time on one frequency = τ_f

Type of response: Impulsive [a]

Crest factor: Not applicable

Formulas for rejection factors: [b]

$$OTR = 10 \log (B_i/B_r)$$

$$OFR_j = -\widetilde{E}_i\{\Delta f_j\}$$

$$SDR = 10 \log (B_r\tau_f)^{-1}$$

Suggested INR threshold: $INR_t = (S/N)_r - (S/\hat{I})_r - PF$ See Equation 16

Notes: [a] Impulses have random amplitude and spacing.

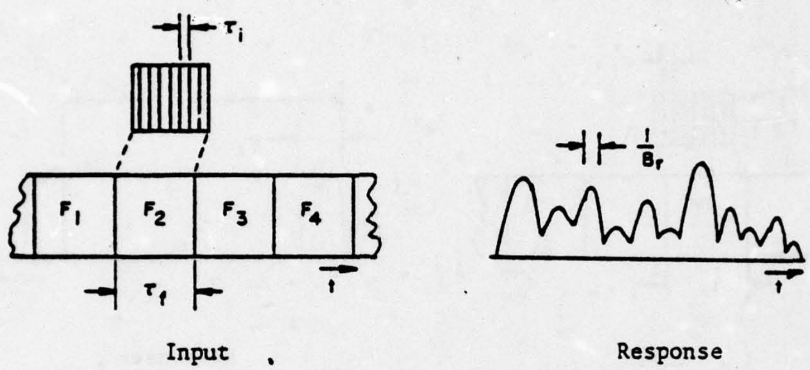
[b] Using these factors in the INR equation yields the expected INR of the peaks of the impulses produced by the jth frequency of the SS signal.

TABLE 6-e

RESPONSE CHARACTERISTICS AND FORMULAS
(FREQUENCY-HOPPED SIGNALS)

Signal/bandwidth condition:

$(B_i\tau_f > 1)$, $(B_i/B_r > 1)$, $(B_r\tau_f < 1)$, $(B_rT_f < 1)$, and $(|\Delta f_j| \geq 0)$



Envelopes of Waveforms

Signal continuity: Dwell time on one frequency = τ_f

Type of response: Noise-like

Crest factor: ≈ 8 dB

Formulas for rejection factors: [a]

$$OTR = 10 \log (B_i/B_r)$$

$$OFR_j = -\tilde{E}_i\{\Delta f_j\}$$

$$SDR = 10 \log N_f$$

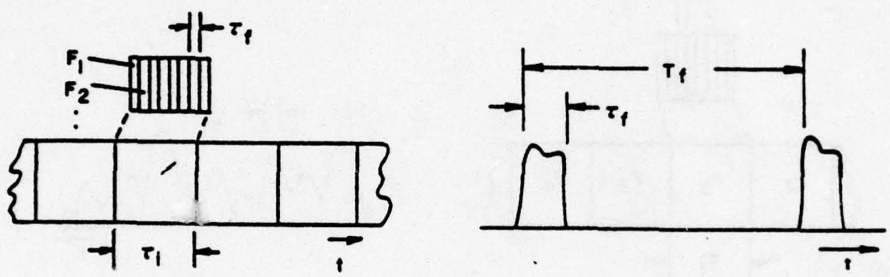
Suggested INR threshold: $INR_t = 10 \log (-1 + 10^{C/10})$

Note [a]Using these factors in the INR equation yield the INR of the response produced by the jth frequency.

TABLE 6-f

RESPONSE CHARACTERISTICS AND FORMULAS
(FREQUENCY-HOPPED SIGNALS)

Signal/bandwidth condition:
$(B_i \tau_f < 1)$, $(B_r \tau_f > 1)$, and $(|\Delta f_j| \leq B_r/2)$



Input Response

Envelopes of Waveforms

Signal continuity: Dwell time on one frequency = τ_f

Type of response: Undistorted [a]

Crest factor: ≈0 dB

Formulas for rejection factors: [b]

$$OTR = 0$$

$$OFR_j = 0$$

$$SDR = 0$$

Suggested INR threshold: $INR_t = (S/N)_r - (S/I_{ud})$. See Equation 20

Notes: [a]The pulses have pseudorandom spacing.

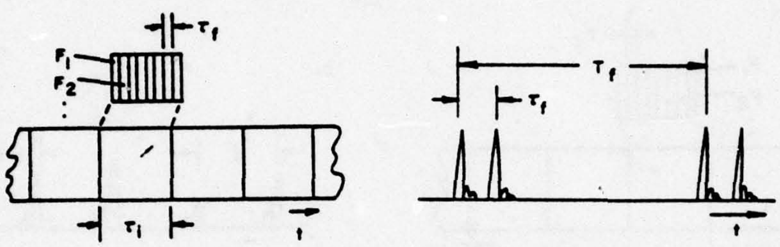
[b]Using these factors in the INR equation yields the INR of the response during the dwell time τ_f produced by the jth frequency of the SS signal.

TABLE 6-g

RESPONSE CHARACTERISTICS AND FORMULAS (FREQUENCY-HOPPED SIGNALS)

Signal/bandwidth condition:

$(B_i \tau_f < 1)$, $(B_r \tau_f > 1)$, and $(|\Delta f_j| > B_r/2)$



Input — Response[c]

Envelopes of Waveforms

Signal continuity: Dwell time on one frequency = τ_f

Type of response: Impulsive,[a] or undistorted[c]

Crest factor: Not applicable

Formulas for rejection factors:[b]

$$OTR = 0$$

$$OFR_j = \text{Max}\,[0,\ R_I\{\Delta f_j\}]$$

$$R_I\{\Delta f_j\} = -20 \log \left\{ \text{antilog}\left[\frac{\tilde{E}_f\{\Delta f_j\} + C_{wb}}{20}\right] + \text{antilog}\left[\frac{-R_r\{\Delta f_j\}}{20}\right] \right\}$$

$$\tilde{E}_f\{\Delta f_j\} = \tilde{F}_f\{\Delta f_j\} - R_t\{\Delta f_o\}$$

$$SDR = 0$$

Suggested INR threshold: $INR_t = (S/N)_r - (\hat{S/I})_r - PF$ (See Equation 16)

Notes:

[a]Impulses resulting from any one frequency have uniform amplitude and pseudorandom spacing.

[b]Using these factors in the INR equation yields the INR of the peaks of the impulses produced by the jth frequency of the SS signal.

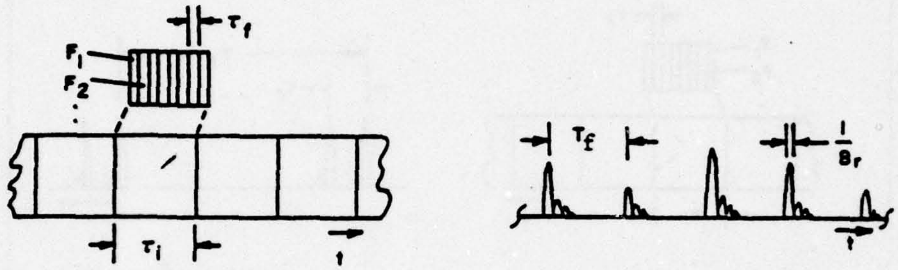
[c]When $R_r\{\Delta f_j\} < -(C_{wb} + F_f\{\Delta f_j\})$, the response waveform is essentially undistorted, as in TABLE 5a.

TABLE 6-h

RESPONSE CHARACTERISTICS AND FORMULAS
(FREQUENCY-HOPPED SIGNALS)

Signal/bandwidth condition:

$(B_i \tau_f < 1)$, $(B_r \tau_f < 1)$, $(B_r T_f > 1)$, and $(|\Delta f_j| \geq 0)$



Input Response

Envelopes of Waveforms

Signal continuity: Dwell time on one frequency = τ_f

Type of response: Impulsive [a]

Crest factor: Not applicable

Formulas for rejection factors: [b]

$$OTR = 10 \log (B_r \tau_f)^{-2}$$

$$OFR_j = -\tilde{E}_f\{\Delta f_j\}$$

$$SDR = 0$$

Suggested INR threshold: $INR_t = (S/N)_r - (S/\hat{I})_r - PF$. See Equation 16

Notes: [a] The impulses resulting from any one frequency have uniform amplitude and pseudorandom spacing.

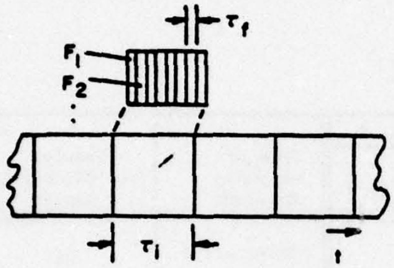
[b] When used in the INR equation, these factors yield the INR of the peaks of the impulses produced by the jth frequency of the SS signal.

TABLE 6-i

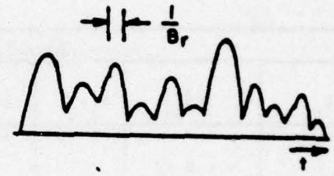
RESPONSE CHARACTERISTICS AND FORMULAS
(FREQUENCY-HOPPED SIGNALS)

Signal/bandwidth condition:

$(B_i\tau_f < 1)$, $(B_r\tau_f < 1)$, $(B_rT_f < 1)$, and $(|\Delta f_j| \geq 0)$



Input



Response

Envelopes of Waveforms

Signal continuity: Dwell time on one frequency = τ_f

Type of response: Noise-like

Crest factor: ≈8 dB

Formulas for rejection factors: [a]

$$OTR = 10 \log \left[N_f/(B_r^2\tau_f)\right]$$

$$OFR_j = -\tilde{E}_f\{\Delta f_j\}$$

$$SDR = 0$$

Suggested INR threshold: $INR_t = 10 \log (-1 + 10^{C/10})$

Note: [a]When used in the INR equation these factors yield the INR of the response produced by the jth frequency of the SS signal.

TABLE 7

FORMULAS FOR REJECTION FACTORS
USED IN THE INR EQUATION

(HYBRID SIGNAL: FREQ-HOPPING/DIRECT SEQUENCE
OR FREQ-HOPPING/PULSED FM)

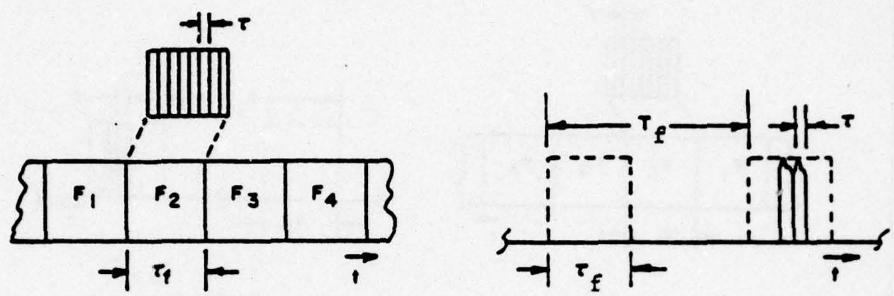
Signal/Bandwidth Conditions [a]					Type of Response Waveform	Formulas (see following tables)
B_t/B_r	$\|\Delta f_j\|$	$B_r\tau_f$	B_rT_f	B_t/B_i		
$* < c_1$	$* \leq B_r/2$				Undistorted	8a
"	$* > B_r/2$				Impulsive, or Undistorted	8b
$c_1 < * < N_c$	$* \geq 0$	$* > 1$			Noise-like	8c
"	"	$* < 1$	$* > 1$		Impulsive	8d
"	"	"	$* < 1$		Noise-like	8e
$* > N_c$	"	"	"	$* \leq N_c$	Noise-like	8f

For each of these entries, substitute the column heading (B_t/B_r, Δf_j, etc.) in place of the star (*) printed in the columns.

TABLE 8-a

RESPONSE CHARACTERISTICS AND FORMULAS
(HYBRID SIGNALS: FH/DS OR FH/PFM)

Signal/bandwidth condition:
$(B_t/B_r < c_1)$ and $(|\Delta f_j| \leq B_r/2)$



Input Response

Envelopes of Waveforms

Signal continuity: Dwell time on one frequency = τ_f

Type of response: Undistorted

Crest factor: ≈0 dB

Formulas for rejection factors:[a]

OTR = 0

OFR = 0

SDR = 0

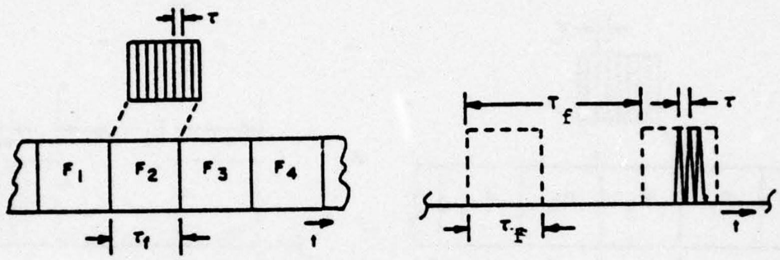
Suggested INR threshold: $INR_t = (S/N)_r - (S/I_{ud})$. See Equation 20

Note: [a]Using these factors in the INR equation yields the INR of the response during the dwell time τ_f produced by the jth frequency.

TABLE 8-b

RESPONSE CHARACTERISTICS AND FORMULAS
(HYBRID SIGNALS: FH/DS OR FH/PFM)

Signal/bandwidth condition:
$(B_t/B_r < c_1)$ and $(|\Delta f_j| > B_r/2)$



Input **Response**[c]

Envelopes of Waveforms

Signal continuity: Dwell time on one frequency = τ_f

Type of response: Impulsive,[a] or undistorted[c]

Crest factor: Not applicable

Formulas for rejection factors:[b]

$$OTR = 0$$

$$OFR_j = \text{Max}\ [0,\ R_I\{\Delta f_j\}]$$

$$R_I\{\Delta f_j\} = -20 \log \left\{ \text{antilog} \left[\frac{\tilde{E}_h\{\Delta f_j\} + C_{wb}}{20} \right] + \text{antilog} \left[\frac{-R_r\{\Delta f_j\}}{20} \right] \right\}$$

$$\tilde{E}_h\{\Delta f_j\} = \tilde{F}_h\{\Delta f_j\} - R_t\{\Delta f_o\} \qquad SDR = 0$$

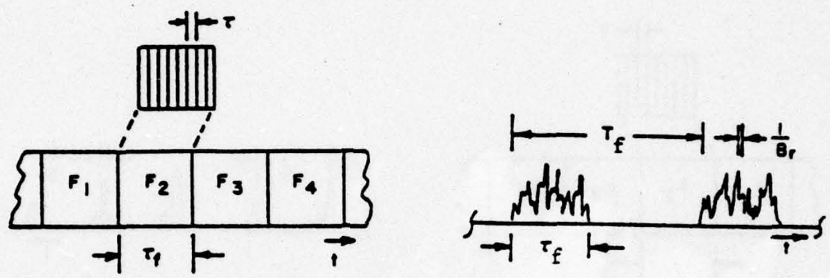
Suggested INR threshold: $INR_t = (S/N)_r - (S/\hat{I})_r - PF$ See Equation 16

Notes:
[a] Except for the leading and trailing impulse of a burst, the impulses due to any one frequency have discrete amplitudes and random spacing.
[b] When used in the INR equation, the factors yield the INR of the peaks of the impulses produced by the jth frequency of the SS signal.
[c] When $R_r\{\Delta f_j\} < -(C_{wb} + \tilde{F}_h\{\Delta f_j\})$, the response waveform is essentially undistorted as in TABLE 6-a.

TABLE 8-c

RESPONSE CHARACTERISTICS AND FORMULAS
(HYBRID SIGNALS: FH/DS OR FH/PFM)

Signal/bandwidth condition:
$(c_1 < B_t/B_r < N_c)$, $(|\Delta f_j| \geq 0)$, and $(B_r \tau_f > 1)$



Input **Response**

Envelopes of Waveforms

Signal continuity: Dwell time on one frequency = τ_f

Type of response: Noise-like

Crest factor: ≈8 dB

Formulas for rejection factors: [a]

$$\mathrm{OTR} = -S_p + 10 \log (B_t/B_r)$$

$$\mathrm{OFR}_j = -\tilde{E}_h\{\Delta f_j\}$$

$$\mathrm{SDR} = 0$$

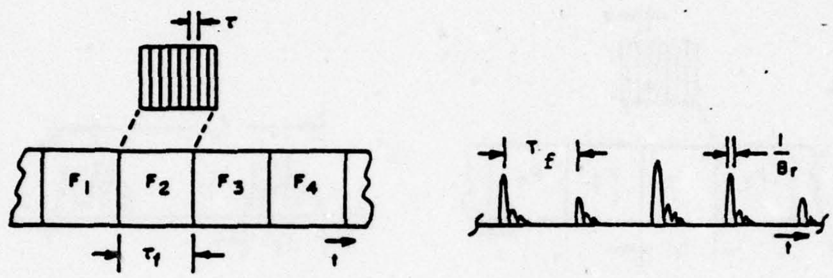
Suggested INR threshold: $\mathrm{INR}_t = 10 \log (-1 + 10^{C/10})$

Note [a]When used in the INR equation, these factors yield the INR of the response during the dwell time τ_f produced by the jth frequency of the SS signal.

TABLE 8-d

RESPONSE CHARACTERISTICS AND FORMULAS
(HYBRID SIGNALS: FH/DS OR FH/PFM)

Signal/bandwidth condition:
$(c_1 < B_t/B_r < N_c)$, $(|\Delta f_j| \geq 0)$, $(B_r\tau_f < 1)$, and $(B_rT_f > 1)$



Input Response

Envelopes of Waveforms

Signal continuity: Dwell time on one frequency = τ_f

Type of response: Impulsive [a]

Crest factor: Not applicable

Formulas for rejection factors: [b]

$$OTR = -S_p + 10 \log (B_t/B_r)$$

$$OFR_j = -\tilde{E}_h\{\Delta f_j\}$$

$$SDR = 10 \log (B_r\tau_f)^{-1}$$

Suggested INR threshold: $INR_t = (S/N)_r - (\hat{S/I})_r - PF$. (See Equation 16)

Notes:
[a]The impulses resulting from any one frequency have pseudorandom amplitudes and spacing.

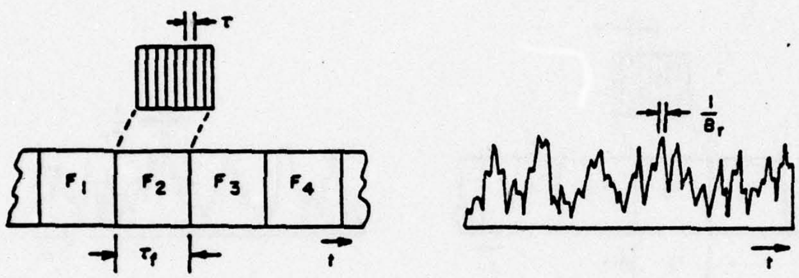
[b]When used in the INR equation, these factors yield the expected INR of the peaks of the impulses produced by the jth frequency of the SS signal.

TABLE 8-e

RESPONSE CHARACTERISTICS AND FORMULAS
(HYBRID SIGNALS: FH/DS OR FH/PFM)

Signal/bandwidth condition:

$(C_1 < B_t/B_r < N_c)$, $(|\Delta f| \geq 0)$, $(B_r\tau_f < 1)$, and $(B_r T_f < 1)$



Input Response

Envelopes of Waveforms

Signal continuity: Dwell time on one frequency = τ_f

Type of response: Noise-like

Crest factor: ≈8 dB

Formulas for rejection factors: [a]

$$\text{OTR} = -S_p + 10 \log (B_t/B_r)$$

$$\text{OFR}_j = -\tilde{E}_h\{\Delta f_j\}$$

$$\text{SDR} = 10 \log N_f$$

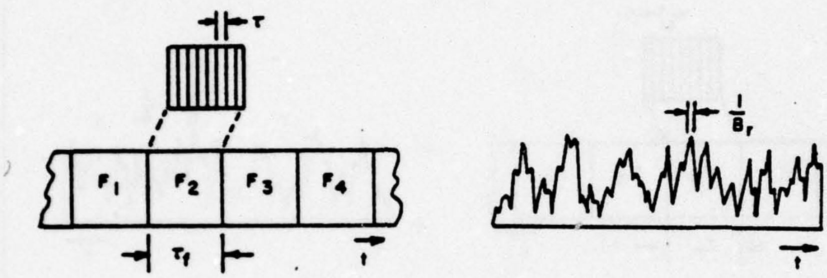
Suggested INR threshold: $\text{INR}_t = 10 \log (-1 + 10^{C/10})$

Note: [a] When used in the INR equation, these factors yield the INR of the response produced by the jth frequency of the SS signal.

TABLE 8-f

RESPONSE CHARACTERISTICS AND FORMULAS
(HYBRID SIGNALS: FH/DS OR FH/PFM)

Signal/bandwidth condition:
$(B_t/B_r > N_c)$, $(|\Delta f_j| \geq 0)$, $(B_t T_f < 1)$, and $(B_{tf}/B_i < N_c)$



Input **Response**

Envelopes of Waveforms

Signal continuity: Dwell time on one frequency = τ_f

Type of response: Noise-like

Crest factor: ≈8 dB

Formulas for rejection factors:

$$\mathrm{OTR} = -S_p + 0 \log N_c$$

$$\mathrm{OFR}_j = -\tilde{E}_h\{\Delta f_j\}$$

$$\mathrm{SDR} = 10 \log N_f$$

Suggested INR threshold: $\mathrm{INR}_t = 10 \log (-1 + 10^{C/10})$

Note: [a] When used in the INR equation, these factors yield the INR of the response produced by the jth frequency.

Explanation of Parameters Used

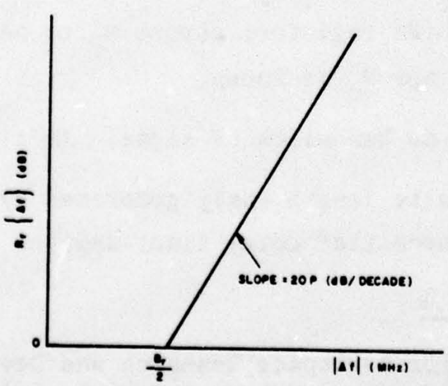
To aid the reader, the parameters used in the tables and formulas in this section are explained in the following paragraphs.

Receiver Parameters. These parameters are G_r, N_r, B_r, which have been defined previously. The formulas used when ($B_t/B_r > c_1 \approx$ 1 to 2) were derived on the assumption that the IF-amplifier filter was of the fourth order or higher, in which case the selectiviy can be approximated by a rectangular function without introducing an error of more than about 1 dB.

The formulas that apply when ($B_t/B_r < c_1 \approx$ 1 to 2) were derived on the assumption that the insertion loss of the IF amplifier is represented by the function

$$R_r\{\Delta f\} = \begin{cases} 0, \text{ for } |\Delta f| \leq B_r/2 \\ 20 \text{ p } \log(2\Delta f/B_r), \text{ otherwise} \end{cases} \quad (3)$$

where p is the order or number of sections in the filter. If p is not specified, assume a value of 4. Figure 6 is a plot of the function.



(Log Scale)

Figure 6. Insertion loss of IF amplifier of victim receiver.

The SS transmitter and Signal Parameters. The transmitter parameters P_t and G_t have been defined previously, and the signal parameters are explained next. Detailed mathematical expressions for the SS signals are not required, and thus, are not given here. Such expression are available in the literature.[4]

TIME WAVEFORM PARAMETERS

The type of modulation employed in the SS signal (PSK, QPSK, OQPSK, MSK, PFM, frequency hopping) and the following parameters must be known:

B_c = chip or clock rate (Mc/s)

τ = chip length (μs); is equal to $1/B_c$; does not apply to PFM.

δ = effective rise/fall time (μs); when the value of δ is not available, a value can be assumed as explained later; this parameter is not required for MSK.

N_c = length of coding sequence (total number of chips); see APPENDIX F for a discussion of this parameter; $N_c = 2^n - 1$ where n is the number of bits in the shift register; assume N_c to be infinite if neither n nor N_c is known.

B_3 = 3 dB bandwidth of signal (MHz); applies to PFM only.

τ_s = pulse length (μs); generated by dispersive or polychromatic[5] delay line; applies to PFM only.

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[4]Advisory Group for Aerospace Research and Development, *Spread Spectrum Communications,* National Technical Information Service U.S. Dept. of Commerce, AD-766,914, July, 1973.

[5]Devore, W. J., "Computer-Aided Spread-Spectrum Signal Design" *IEEE Transaction on Communications.,* Vol. COM-25, No. 8, Aug. 1977, p. 861-867.

The *rise/fall time* (δ) for PFM signals pertains to the 0-to-100% rise/fall time of the pulses produced by the dispersive delay line that is used to spread the bandwidth; δ is generally specified for PFM signals.

For PSK, QPSK, and OQPSK, δ is assumed here to be 1/2 the time required for the phase to change from one state to another. Therefore δ denotes the effective rise time. In the literature, it is commonly assumed that the phase changes occur instantly. Such an assumption is all right when one is primarily concerned with the in-band region of the signal spectrum, but for purposes of EMC analyses a more realistic representation of the effects of the phase changes is desirable. For example, either the phase function or the amplitude function can be assumed to be trapezoidal with a rise/fall time of δ. The spectrum resulting from either type of function has almost the same bounds. For mathematical convenience, the trapezoidal amplitude function is assumed for PSK, QPSK, and OQPSK signals. Unless a known value is available, δ can be assumed to be $1/(10\ B_c)$ for PSK, QPSK, and OQPSK.

MSK signals are generated using sinusoidal pulses rather than trapezoidal, so that the parameter δ does not apply to them.

GATING PARAMETERS

If the SS signal is transmitted in bursts, as depicted in Figure 7, the following must be known:

τ_g = burst or gate length, (μs)

T_g = average burst or gate spacing, (μs)

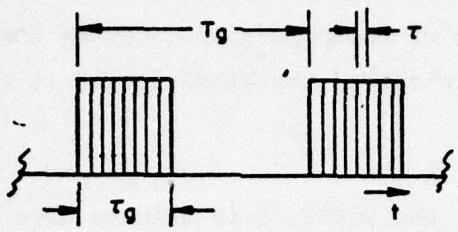


Figure 7. Gating parameters.

FREQUENCY HOPPING PARAMETERS

For frequency-hopping signals as depicted in Figure 8, the following must be known:

τ_f = dwell time on one frequency (μs)

B_h = frequency-hopping rate, megahops/second (Mh/s), $B_h = 1/\tau_f$

T_f = average time for sequencing through all frequencies used (μs), $T_f = N_f \tau_f$

N_f = number of frequencies used

F_j = (j = 1, 2, 3, . . . N_f) frequencies used

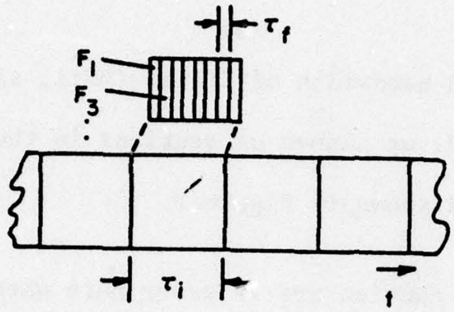
B_i = bandwidth of information or baseband signal (MHz); required only if $B_i \tau_f > 1$

The information or baseband signal is assumed to be in digital form having a chip length of $\tau_i = 1/B_i$.

TRANSMITTER FILTER PARAMETERS

When the SS transmitter employs a bandpass filter to attenuate out-of-band emissions, the insertion loss is given by

(a)



(b)

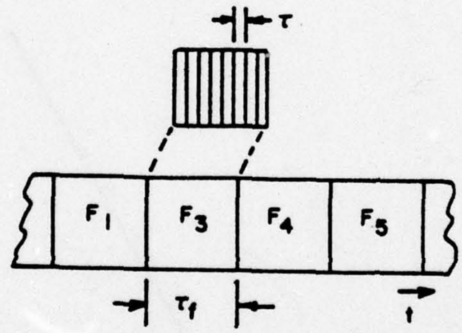


Figure 8. Frequency-hopping parameters (a) when $\tau_i > \tau_f$, and (b) when $\tau_i < \tau_f$.

$$R_t\{\Delta f\} = \begin{cases} 0, \text{ for } |\Delta f| \leq B_F/2 \\ 20\ p \log(2\Delta f/B_F), \text{ otherwise} \end{cases} \quad (4)$$

where

B_F = 3 dB bandwidth of filter (MHz), always $\geq B_t$

p = order or number of sections in the filter

A plot of $R_t\{\Delta f\}$ is shown in Figure 9.

Some words of caution are in order here when PSK, QPSK, or offset-QPSK signals are involved. If the rise/fall time of the filtered,

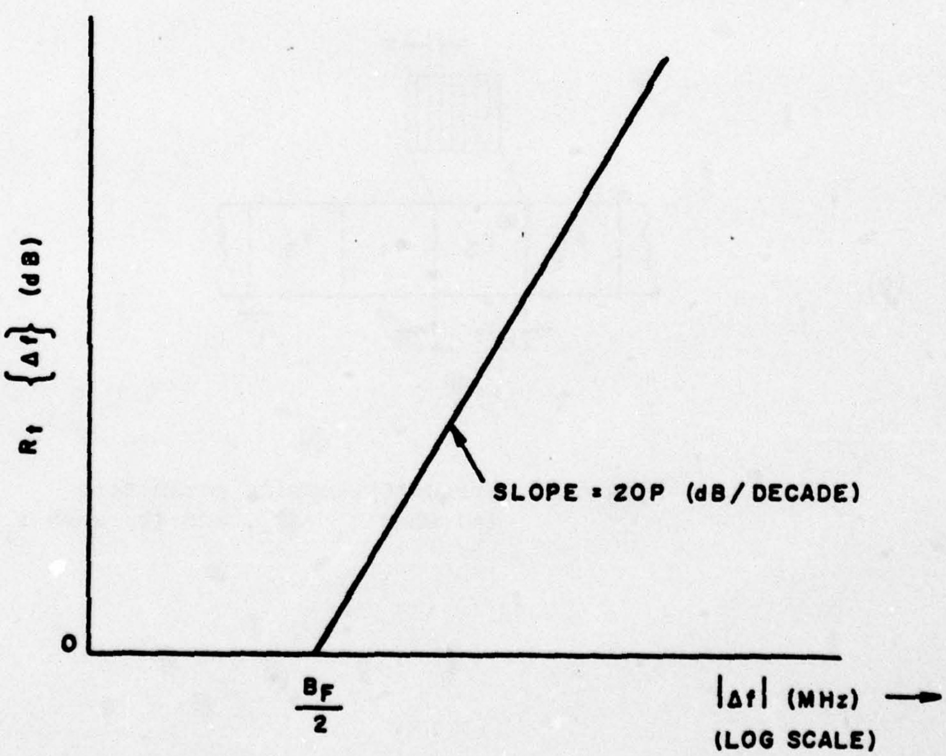


Figure 9. SS transmitter-filter insertion loss.

or radiated, signal is known and is used in calculating the spectral bounds, $\widetilde{F}_t\{\Delta f\}$, the effects of the filter are included in $\widetilde{F}_t\{\Delta f\}$ and there is no need to subtract the insertion loss $R_t\{\Delta f\}$. If, however, the rise/fall time were assumed to be $\delta = (10\ B_c)^{-1}$, as suggested previously for cases where a known value is not available, the filter insertion loss should be accounted for, i.e., $\widetilde{F}_t\{\Delta f\} - R_t\{\Delta f\}$ should be used to represent the bounds on the radiated spectrum.

SIGNAL SPECTRUM PARAMETERS AND FUNCTIONS

The parameters S_p, B_t, c_1, and F_n are obtained as follows:

The *spectrum peaking factor* (S_p), a term used in calculating OTR, is the peak of the normalized spectrum (normalized to p_t/B_t) and is given by

$$S_p = \begin{cases} 0 \text{ dB, for PSK, QPSK, or PFM} \\ 2\text{, for MSK, or continuous PSK} \\ 3\text{, for OQPSK} \end{cases} \qquad (5)$$

The *signal bandwidth* (B_t)[a] is defined here as

$$B_t = \begin{cases} B_c \text{ for DS techniques (PSK, QPSK, OQPSK, MSK)} \\ B_3 \text{ for PFM technique} \end{cases} \qquad (6)$$

where B_c and B_3 have been defined previously. The PSK, OQPSK, and MSK signals have one bit/chip; whereas, the QPSK signals have two bits/chip.

[a]The bandwidth (B_t) is not the same as the bandwidth denoted by BW_{RF} in Reference 1. $BW_{RF} = 2B_c$ for PSK or QPSK.

The *parameter* c_l, which is used to define the limits of several bandwidth conditions is given by

$$c_1 = \text{antilog } (S_p/10) \approx 1 \text{ to } 2 \qquad (7)$$

The *transmitter spectral noise floor* (F_n), which is included in the signal spectrum is

$$F_n = (N/C) + 60 - S_p + 10 \log B_t \qquad (8)$$

where N/C, which is a property of the transmitter, is the spectral power density (mW in 1 Hz) of the noise normalized to the carrier power (mW), expressed in dB. Both the above formula and the noise performance of microwave transmitters are discussed in APPENDIX C.

If a value for N/C is unavailable, assume that

$$N/C = -140 \text{ dB/Hz}$$

As shown in Figures 10 through 12 the noise floor is represented by a constant level for large values of $|\Delta f|$. However, because of the frequency response of the antenna and microwave components, as well as the transmitter filter, the noise floor in the emission spectrum is expected to fall off at a rate of 20 dB/decade or more for large values of $|\Delta f|$. If the transmitter does not have a band-pass filter, it is suggested that the noise floor be given a 20 dB/decade slope.

Frequency separation parameters used in calculating OFR are defined as follows:

$$\Delta f = f_t - f_r \qquad (9)$$

where f_t is the carrier frequency of the SS signal (no frequency hopping) and f_r is the tuned frequency of the victim receiver.

Then $$\Delta f_j = F_j - f_r \tag{10}$$

where F_j is the jth frequency in a frequency-hopped SS signal,

and $$\Delta f_o = F_o - f_r \tag{11}$$

where F_o is the center frequency of the transmitter's bandpass filter in a frequency-hopped SS system.

Spectral Functions and Bounds

The emission spectrum $E_t\{\Delta f\}$ is the filtered signal spectrum, that is

$$E_t\{\Delta f\} = F_t\{\Delta f\} - R_t\{\Delta f\} \tag{12}$$

where $F_t\{\Delta f\}$ is the power-density spectrum of the signal and $R_t\{\Delta f\}$ is the insertion loss of the bandpass filter in the transmitter. The bounds on the emission spectrum are given by

$$\tilde{E}_t\{\Delta f\} = \tilde{F}_t\{\Delta f\} - R_t\{\Delta f\} \tag{13}$$

The bounds on the signal spectrum can be obtained either graphically or mathematically using the appropriate procedures and equations given in Figures 10, 11, or 12, depending on the type of modulation. The insertion loss $R_t\{\Delta f\}$, is obtained using Equation 4. Using plots of $\tilde{F}_t\{\Delta f\}$ and $R_t\{\Delta f\}$ to obtain a plot of $\tilde{E}_t\{\Delta f\}$ was illustrated in Figure 3.

For frequency-hopped signals, the spectral bounds $\tilde{F}_i\{\Delta fj\}$, and $\widetilde{F}_h\{\Delta f_j\}$ can be obtained using Figures 10, 11, and 12, with certain substitutions, as explained in APPENDIX F.

Additional information on the spectral functions and bounds is given in APPENDIX F. This is used only when Option 3 is used to calculate OFR.

Spectrum Constant, C_{wb}

The constant C_{wb}, which is used in the formulas for $R_I\{\Delta f\}$ in TABLES 4, 6, and 8 is obtained by use of an appropriate formula from TABLE 9.

TABLE 9

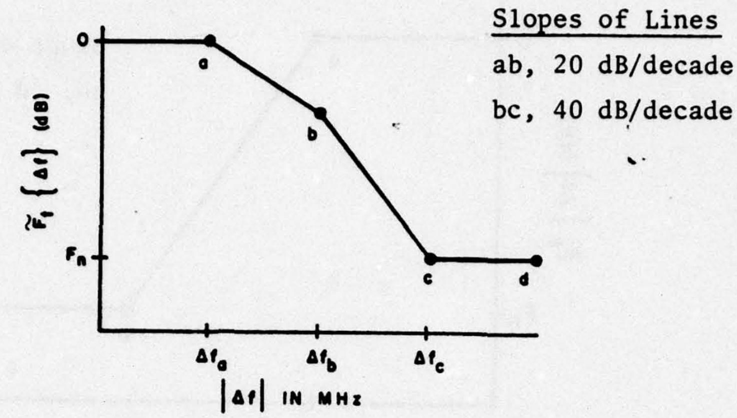
FORMULAS FOR OBTAINING C_{wb}

Type of Modulation	C_{wb}
PSK, and QPSK	$20 \log (B_r/B_c)$
OQPSK	$20 \log(\sqrt{2}\, B_r/B_c)$
MSK	$20 \log \frac{4}{\pi} \frac{B_r}{B_c}$

INR THRESHOLD

To determine the compatibility of an SS transmitter and a conventional receiver, the INR is calculated using an appropriate formula from TABLE 2, 4, 6, or 8 and compared with some predetermined threshold value (INR_t).

In choosing a value for INR_t, one of the factors that must be



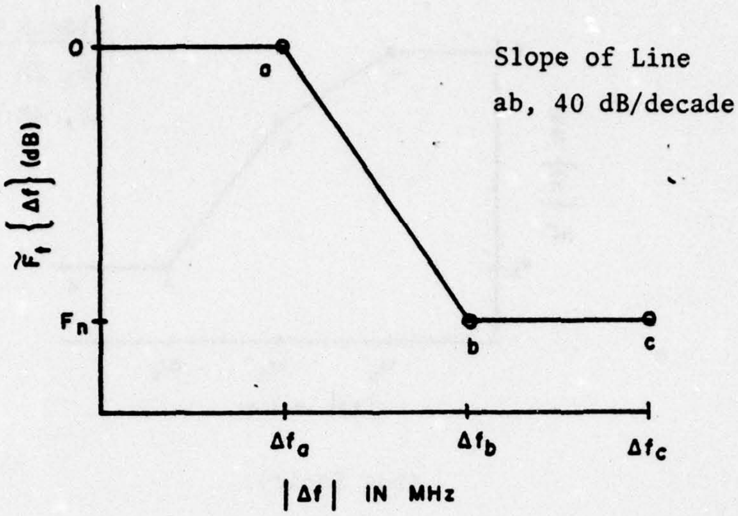
(Log Scale)

$$\Delta f_a = \begin{cases} B_c/\pi, \text{ for PSK, QPSK} \\ B_c/(2\pi), \text{ for OQPSK} \end{cases} \qquad \Delta F_b = 1/(2\pi\delta)$$

$$\Delta f_c = \text{antilog}\left[(^1/_2 \log \Delta f_a \Delta f_b) - F_n/40)\right]$$

$$\tilde{F}_t\{\Delta f\} = \begin{cases} 0, & |\Delta f| \le \Delta f_a \\ 20 \log(\Delta f_a/\Delta f), & \Delta f_a < |\Delta f| \le \Delta f_b \\ 20 \log(\Delta f_a \Delta f_b/\Delta f2), & \Delta f_b < |\Delta f| \le \Delta f_c \\ F_n, & |\Delta f| > \Delta f_c \end{cases} \quad \underline{\text{when}}$$

Figure 10. Spectral bounds, $\tilde{F}_t\{\Delta f\}$, for PSK, QPSK, and OQPSK signals.



(Log Scale)

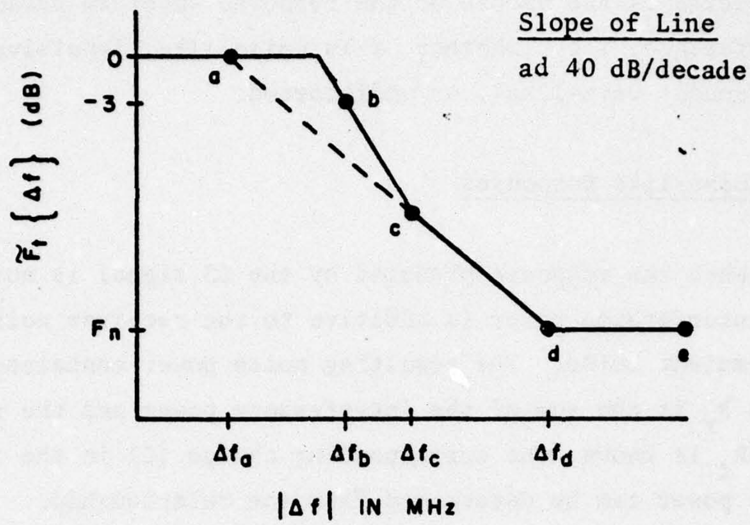
$$\Delta f_a = B_c/4$$

$$\Delta f_b = \text{antilog}\left[(\log\Delta f_a) - (F_n/40)\right]$$

$$\tilde{F}_t\{\Delta f\} = \begin{cases} 0, & |\Delta f| \le \Delta f_a \\ 40\log(\Delta f_a/\Delta f), & \Delta f_a < |\Delta f| < \Delta f_b \\ F_n, & |\Delta f| > \Delta f_b \end{cases} \quad \underline{\text{when}}$$

Note: The spectrum exceeds this bound by as much as 2 dB for $\Delta f_a < |\Delta f| < 2.5\ B_c$, but is always less than the bound for $|\Delta f| \ge 2.5\ B_c$

Figure 11. Spectral bounds, $\tilde{F}_t\{\Delta f\}$, for MSK signals.



(Log Scale)

$\Delta f_a = \frac{1}{\pi}\left(\frac{B_t}{\tau_s}\right)^{\frac{1}{4}} \left(\frac{1}{\delta}\right)^{\frac{1}{2}}$ $\qquad$ $\Delta f_b = B_t/2$

$\Delta f_c = B_t$ $\qquad$ $\Delta f_d = \text{antilog}\left[(\log\Delta f_a) - F_n/40\right]$

Line bc is drawn through points b and c.

$\tilde{F}_t\{\Delta f\}$ = this function is obtained most conveniently using the graph.

Note: The procedures given here are for $\delta \ll \tau_s$ and $(1/\pi\delta) \leq B_t$. See Reference 20 for $(1/\pi\delta) > B_t$.

Figure 12. Spectral bounds, $\tilde{F}_t\{\Delta f\}$, for PFM signals.

considered is the nature of the response waveform produced by the interference; i.e., whether it is noise-like, impulsive, CW-like (continuous-wave-like), or undistorted.

For Noise-like Responses

When the response produced by the SS signal is noise-like, the interference power is additive to the receiver noise, including any ambient noise. The resulting noise power contained within bandwidth B_r is the sum of the interference power and the receiver noise. If INR_t is known, the corresponding change (C) in the total receiver noise power can be determined from the relationship.

$$C = 10 \log(1 + 10^{INR_t/10}) \tag{14}$$

Obviously, C is also the corresponding change in the receiver's signal-to-noise ratio.

On the other hand, if the allowable change (C) in receiver noise or signal-to-noise ratio is known, INR_t can be determined by

$$INR_t = 10 \log(-1 + 10^{C/10}) \tag{15}$$

A plot of C as a function of INR_t is shown in Figure 13.

For Response to Impulses

When the interference at the IF-amplifier output is impulsive, the threshold is

$$INR_t = (S/N)_r - (S/\hat{I})_r - PF \tag{16}$$

where $(S/N)_r$, the signal-to-noise ratio, and $(S/\hat{I})_r$, the signal-to-

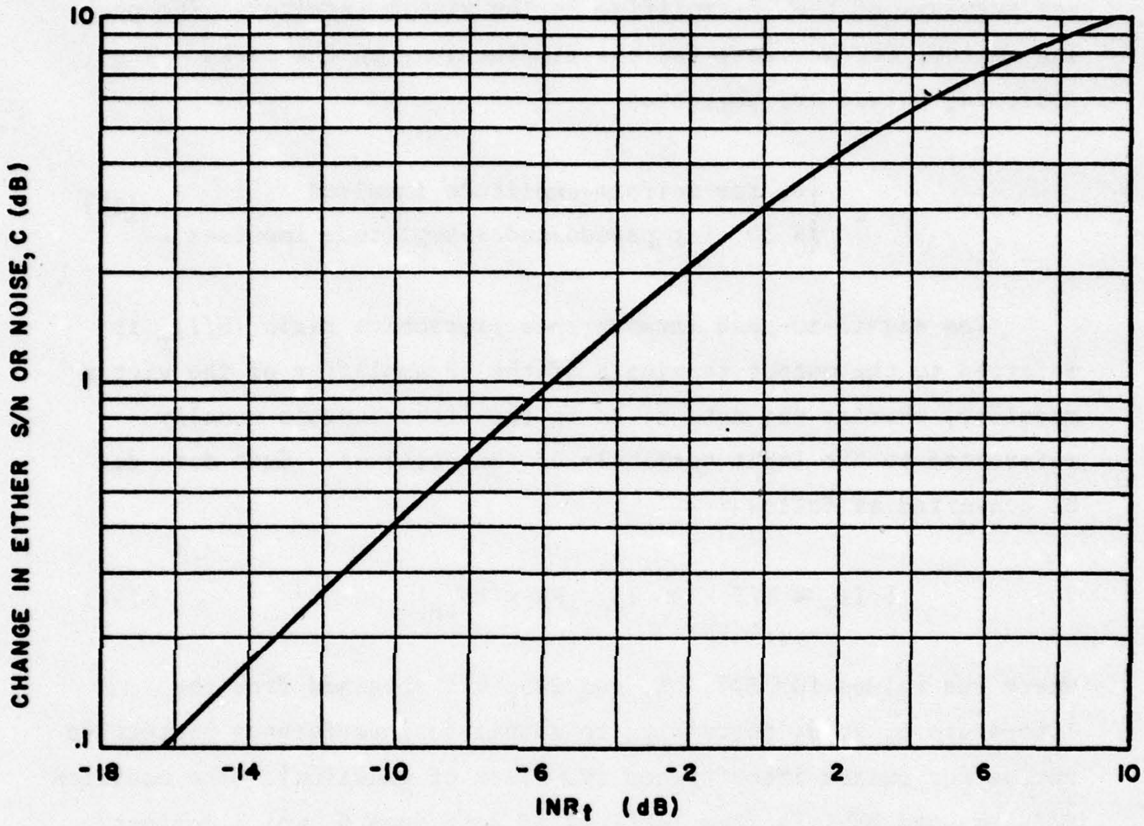


Figure 13. Relationship between INR-threshold and allowable in S/N or receiver noise.

peak interference protective ratio, are both referred to the output terminal of the IF-amplifier in the victim receiver. The peaking factor, PF, accounts for the fluctuations in the peaks. The following values are suggested

$$PF = \begin{cases} 0, \text{ for uniform-amplitude impulses} \\ 8 \text{ db, for pseudorandom-amplitude impulses} \end{cases} \quad (17)$$

The signal-to-peak interference protective ratio $(S/\hat{I})_r$ is referred to the output terminals of the IF amplifier of the victim receiver, whereas the data given in the literature is usually referenced to the input terminals of the receiver. Such data can be converted as follows:

$$(S/\hat{I})_r \approx S/I - 20 \log (PW \times BW_{IF}) \quad (18)$$

where the values for S/I, PW, and BW_{IF} are obtained from the literature 6, 7, 8, pertaining to signal-to-interference protective ratios for pulsed interference (PO class of emission). The notation S/I, PW, and BW_{IF} is from TABLE IV of Reference 6, which defines S/I as the signal-to-interference ratio (peak power of the interference pulses) at the test receiver input, PW is the pulse width of the interference pulse and BW_{IF} is the bandwidth of the IF amplifier of the test receiver.

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[6]"Provisional Signal-to-Interference Protection Ratios Required for Spectrum Utilization Investigations," CCIR, XIIIth Plenary Assembly, Vol. I, *Spectrum Utilization and Monitoring,* ITU, Geneva 1975, Report 525, p. 245-249.

[7]Hernandez, A. A., *Investigation of Pulsed Interference to Narrowband FM receivers,* ESD-TR-75-023, ECAC, Annapolis, MD, December 1975.

[8]ECAC [July, 1973] *Communications/Electronics Receiver Performance Degradation Handbook.* The Frequency Management Support Division, Office of Telecommunication, U.S. Department of Commerce and the Electromagnetic Compatibility Analysis Center (E.C.A.C.), ESD-TR-73-014, NTIS Number AD 764710.

The ratio S/I is expressed in dB. In using Equation 18, PW should be in μs and BW_{IF}, in MHz.

The data given in the literature show that the pulse repetition rate (PRF) also influences the S/I protective ratio. For example, Hernandez (Reference 13) reports that for narrowband FM, varying the PRF of the interference from 400 to 1000 pps increased the "protective ratio" about 5 dB, or that increasing the PFR from 1000 to 4000 pps also requires a 5-dB increase in the S/I protective ratio. In using the data from the literature, one should try to find data obtained with a PRF that is as close as possible to PRF_i, the average pulse repetition rate of the impulsive interference caused by the SS signal. If the PRF associated with the S/I obtained from the literature is much different from PRF_i, the ratio used should be adjusted. One might use Hernandez's results, for example, to judge the degree of adjustment.

An example follows to explain the application of Equation 18. Consider a case in which the victim receiver is frequency-modulated voice (F3) and the required performance is 0.7 AI. TABLE IV of Reference 6 (for an F3( ) desired signal, BW_{IF} = .016 MHz, PO interference, and PW = 5 μs) gives a value of S/I = -24 dB. Equation 18 yields $(S/\hat{I})_r = -24 - 20 \log (5 \times .016) = -2$ dB.

Continuing with this example, assume that $(S/N)_r = 30$ dB and the impulses are of random amplitude so that PF is estimated to be 8 dB. Equation 16 yields $INR_t = 30 - (-2) - 8 = 24$ dB.

For CW-like Responses

When the interference at the IF-amplifier output is CW-like, the threshold is obtained by

$$INR_t = (S/N)_r - (S/I_{cw}) \quad (19)$$

where (S/I_{cw}) is the signal-to-interference power ratios for CW interference, which can be obtained from References 6 and 8.

For Undistorted Responses

When the response is undistorted the threshold is

$$INR_t = (S/N)_r - (S/I_{ud}) \quad (20)$$

where (S/I_{ud}) is the signal-to-interference protection ratio for PSK interference in victim receivers with bandwidths that are equal to or greater than the bit rate of the interference. An example of such a situation is the 1.1F1 interference in TABLE IV of Reference 6. It appears from this reference that S/I_{ud} is generally about 5 dB greater than S/I_{cw} (concluded by comparing the protection ratios required for 1.1F1 and interference).

APPLICATION OF INR EQUATION

The procedure to be used in a particular application depends on whether the SS signal is frequency hopped, as will be explained.

SS Signal With No Frequency Hopping

When the SS signal is not frequency hopped, the procedure in volves the following steps:

1. Determine the signal/bandwidth condition using the limits given in TABLE 2 or 3.

2. Calculate the INR using Equation 1 and the appropriate formulas for the rejection factors given in TABLE 2 or 4.

3. Having determined the signal/bandwidth condition, Step (1), refer to TABLE 1 or 3 to determine the nature of the response waveform (undistorted, noise-like, impulsive, etc.).

4. Determine the approprate INR threshold using Equation 15, 16, 19, or 20, depending on the nature of the response waveform, found in Step 3.

5. Compare the INR calculated in Step 2 with the threshold calculated in Step 4 to see of an unacceptable interference condition should exist.

SS Signal With Frequency Hopping

For a frequency-hopped SS signal, including hybrid systems, the procedure is as follows:

1. Determine the signal/bandwidth condition using the limits given in TABLE 5 or 7.

2. Consider the frequency-hopped signal as consisting of N_f time-shared signals at frequencies F_j, with $j = 1, 2 \ldots, N_f$. Calculate the INR associated with each of the N_f signals, using Equation 1 and the appropriate formulas for the rejection factors given in TABLE 6 or 8.

3. Having determined the signal/bandwidth condition in Step 1, refer to TABLE 5 or 7 to determine the nature of the response waveforms (undistorted, noise-like, impulsive, etc.).

4. Determine the appropriate INR threshold using Equation 15, 16, 19, or 20, depending on the nature of the response waveform, found in Step 3.

5. Compare the INR values calculated in Step 2 with the threshold calculated in Step 4 to see if an unacceptable interference condition is likely to exist.

* * *

When the victim receiver is essentially on-tune with one of the frequencies, as the example in Figure 14, the response to that frequency will predominate (see Figure E-9 in APPENDIX E). For this kind of situation, the SS signal might be treated as a single-frequency signal that is transmitted in bursts of length τ_f and spacing $T_f = N_f \tau_f$.

In some frequency-hopped systems, the frequency spacing is equal to the hopping frequency (B_h), which results in the central lobes of the spectra being configured as shown in Figure 15. Note that whenever the frequency spacing is a multiple of B_h, the center of the central lobe of any one spectrum coincides with the nulls of all of the other spectra. A narrowband receiver tuned exactly to one frequency would be tuned to the nulls of the spectra of the other frequencies and would receive very little power from them.

When the victim receiver is not on tune with any of the frequencies, so that none produces a predominating response, one could take two approaches:

1. determine the INR associated with each frequency and then add them in an appropriate way,[a] or

2. treat the SS signal as an uninterrupted signal at the frequency closest to the receiver tuned frequency.

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[a]Frequency-hopped signals are usually not coherent so that the power in overlapping responses from differenct frequencies can be added.

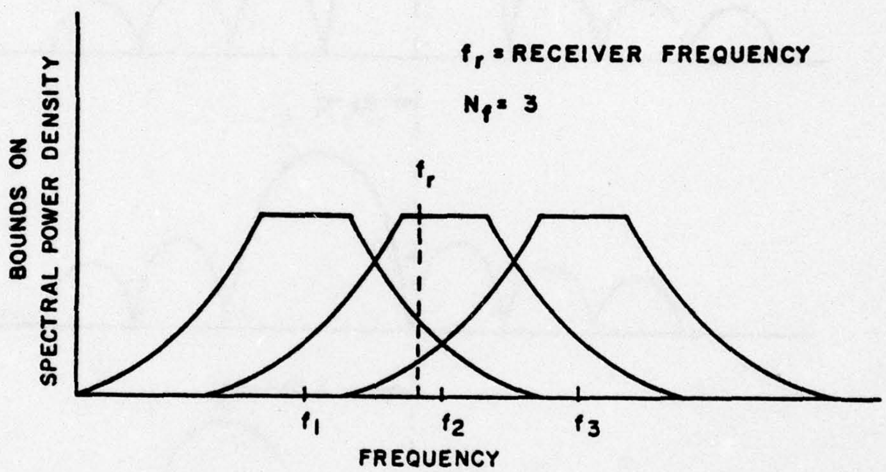

Figure 14. An example of a frequency-hopping signal with three frequencies.

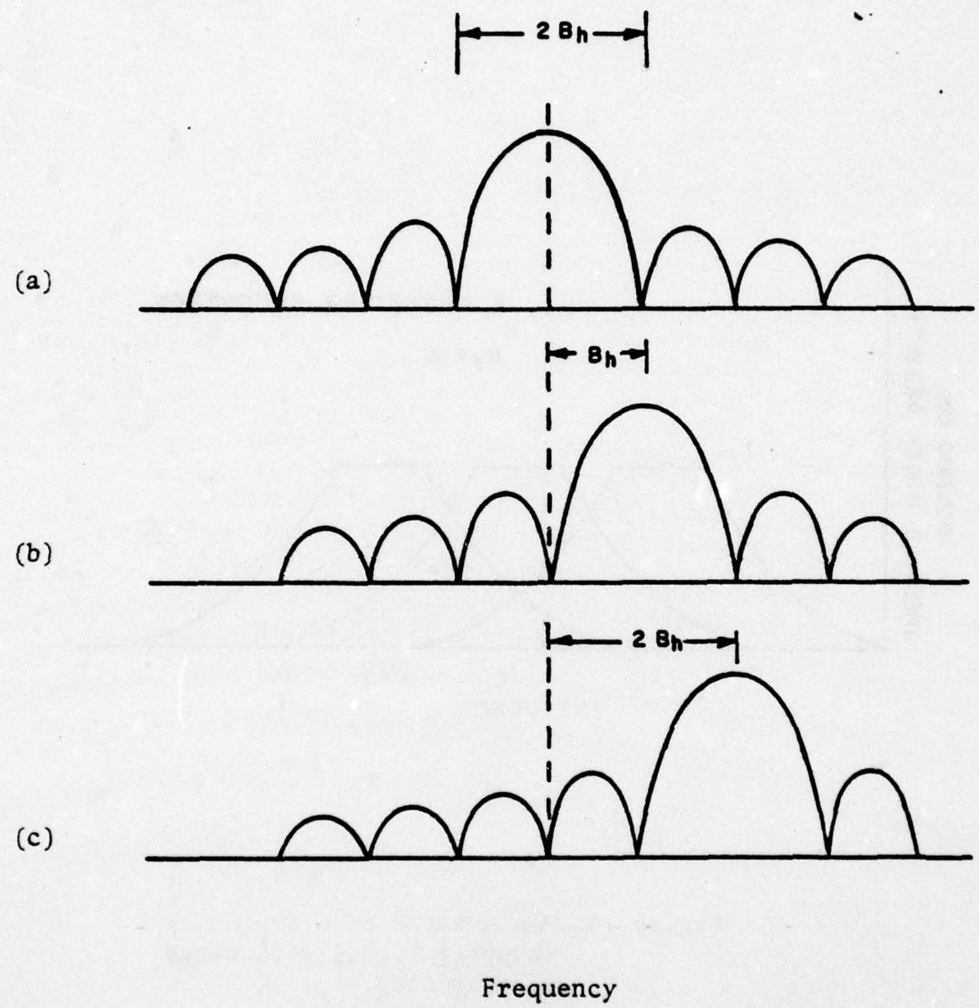

Figure 15. Spectrum configuration when frequencies are separated by hopping rate (B_h): (a), (b), and (c) are the spectra associated with frequencies F_1, F_2, and F_3.

SECTION 3

PARAMETER SENSITIVITIES AND TRADE-OFFS

The INR equation given in the preceding section relates the radio transmission loss, the parameters of the SS transmitter, and the parameters of the conventional receiver and, can therefore, be used to determine the trade-offs between the parameters. To illustrate how this can be done, several examples will be given.

FREQUENCY/DISTANCE SEPARATION TRADE-OFF (When $c_1 < B_t/B_r < N_c$)

Two parameters that can be varied by operations personnel to achieve a compatible operation are: frequency separation, and distance separation. A compatible operation is defined as one in which the INR does not exceed some predetermined threshold (INR_t), that is

$$INR \leq INR_t \tag{21}$$

The INR equation can be written as

$$INR = P_t + G_t + G_r - N_r - L_p - OTR - OFR - SDR \tag{22}$$

where

$$N_r = -114 + F_r + 10 \log B_r$$

$$OTR = 10 \log(B_t/B_r) - S_p$$

F_r = noise figure of receiver, including ambient noise (dB)

The formula for SDR is given in TABLE 2. Combining Expressions 21 and 22 yields

$$(OFR + L_p) \geq A - 10 \log B_t \tag{23}$$

where

$$A = P_t + G_t + G_r + 114 - F_r + S_p - SDR - INR_t \tag{24}$$

The quantity ($OFR + L_p$) in Expression 23, is the minimum required sum of the off-frequency rejection and the radio transmission path loss for a compatible operation; i.e., for $INR \leq INR_t$. The OFR is dependent on the frequency separation (Δf), and L_p is dependent on the distance separation.

For any given value of OFR (and hence frequency separation) there is a particular value of L_p (and hence distance) that yields the minimum required value of ($OFR + L_p$); the corresponding pair of values of frequency separation and distance separations determines the abscissa and ordinate of one point on the frequency/distance separation curve; obviously, the curve can be generated by varying the frequency separation and calculating the required distance separation. The frequency/distance-separation concept and an algorithm for calculating and plotting the curve are explained by Harris.[9] This capability was updated by H. Crosswhite, and a user's manuel was published.[10]

REQUIREMENT FOR COCHANNEL OPERATION (when $c_1 < B_t/B_r < N_c$)

One of the features often expected of SS systems is the ability to operate cochannel with conventional, narrowband radio systems without causing any detectable interference in the narrowband receiver. This requires that

$$L_p \geq A - 10 \log B_t \tag{25}$$

[9]Harris, R., *Off-Frequency Rejection Capabilities*, ECAC-TN-71-25, ECAC, Annapolis, MD, March 1971.

[10]Crosswhite, H., *FDR User's Manual*, ECAC-UM-77-012, ECAC, Annapolis, MD, November 1977.

where A is defined by Equation 24. Expression 25, which is obtained by letting OFR = 0 in Expression 23, reveals the difficulty of cochannel operation of a non-frequency-hopping, continuous transmitter with a nearby conventional, narrowband system. Gonsider the following example involving a direct-sequence PSK SS transmitter and conventional, narrowband receiver with:

P_t = 30 dBm B_t = 20 MHz

G_t = 3 dBi G_r = 3 dBi

S_p = 0 (for PSK)

Transmitter frequency = 2000 MHz

Transmission is uninterrupted, i.e., SDR = 0

B_r = .025 MHz INR_t = 0 dB

F_r = 20 dB

Using Equation 24 gives A = 130, and then using Expression 25 yields

$$L_p \geq 130 - 10 \log 20 = 117 \text{ dB}$$

For a free-space radio transmission path between the SS transmitter and narrow-band receiver, the required distance separation in the above example would be approximately 4.5 nautical miles (nmi).

Next, assume that the SS system in the above example transmits in bursts of length τ_g = 10 μs with a spacing of T_g = 50 μs. The signal-discontinuity rejection would be

$$SDR = 10 \log (B_r \tau_g)^{-1} = 10 \log(.025 \times 10)^{-1} = 6 \text{ dB}.$$

Equation 24 would then give A = 124, and Equation 25 gives L_p = 111 dB.

The corresponding free-space transmission path length would be about 2.25 nmi. Of course, the information transmission rate of the SS system would be reduced to $\tau_g/T_g = .2$, or 20% of the original rate obtained with the continuous transmission. To prevent this reduction of the information transmission rate, frequency hopping with five frequencies could be employed.

APPENDIX A

RESPONSES OF BANDPASS FILTERS TO SPREAD SPECTRUM SIGNALS

GENERAL

In deriving the INR equation that was presented in Section 2, the victim receiver was represented by a passive bandpass filter. The fundamental relationships of the responses of bandpass filters to digital signals were determined in an analysis performed in the off-frequency rejection task of the ECAC model development project. A derivation of the response equations and a FORTRAN program that uses the equations for calculating and plotting the response waveforms are being documented in the final report on the Off-frequency Rejection Task (Reference 2) which will be completed at the end of FY 77.

The general behavior of the response waveforms is highly dependent on the bandwidth ratio B_t/B_r. For purposes of explanation three conditions are considered; these are:

(1) $B_t/B_r << 1$

(2) $1 << B_t/B_r << N_c$

(3) $B_t/B_r >> N_c$

where B_t is the bandwidth of the SS signal, which for a PSK signal is equal to the chip rate B_c; B_r is the 3-dB bandwidth of the bandpass filter (Butterworth or Chebyshev design); and N_c is the total number of chips in the pseudorandom sequence. For maximal-length sequences $N_c = 2^n - 1$, where n is the number of stages in the code-generating shift register.

When $B_t/B_r << 1$

Obviously, with this condition, the input signal passes through the filter with little or no distortion when the signal is on-tune; i.e., when $\Delta f = 0$, where Δf is the difference between the carrier frequency of the input signal and the center frequency of the band-pass filter. An example of the waveform obtained with this condition is Figure E-4-a (APPENDIX E).

When $|\Delta f| \approx B_r/2$, the response is distorted as shown in Figure E-4-b.

When $|\Delta f| >> B_r/2$, the response consists of impulses, on of which occurs each time the phase is shifted, as shown in Figure E-4-d.

When $1 << B_t/B_r << N_c$

When this condition exists, the bandpass filter produces a noise-like response that can be explained by using the results of Neuvo and Ku's analysis[11] of a Gaussian noise source employing a pseudorandom binary sequence (PRBS). Their Gaussian noise source consisted of a PRBS generator followed by an up/down counter, output register, and D/A converter, as shown in Figure A-1. The PRBS generator, consisting of a shift register with feedback, produced a binary sequence of length $2^n - 1$. For mathematical convenience, the shift-register is designed so that the output is (+1, -1) instead of the usual (0, 1). For large n, the output of the PRBS generator is assumed by Neuvo and Ku to be a sample from a binomial distribution with equal probability for +1 and -1.

The up/down counter sums N consecutive samples (Δx) from the PRBS

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[11] Neuvo, Y., and Ku, H., "Analysis and Digital Realization of a Pseudorandom Gaussian and Impulse Noise Source," *IEEE Transaction on Communications.*, Vol. COM-23, p. 849-858, Sept. 1975.

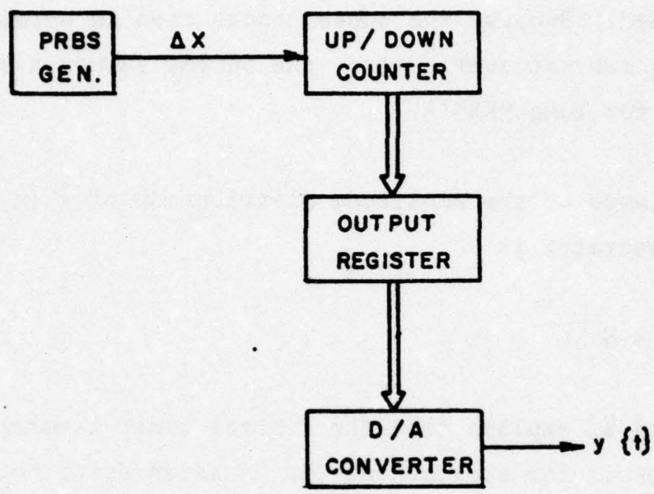

Figure A-1. Gaussian noise source.
(Reference 11).

generator and the result is stored in the output register to produce a sample of x of a Gaussian amplitude distribution. The digital-to-analog converter gives the analog output y(t). To obtain the next value of x, the counter is reset and the next N consecutive samples (Δx) are summed. Because the subsequences used to obtain consecutive output values are not overlapping, the output values x are practically uncorrelated for long PRBS's.

The variance of the amplitude distribution of x in Neuvo and Ku's noise generator is

$$\sigma^2 = N \tag{A-1}$$

Neuvo and Ku explain that the central limit theorem provides the theoretical basis for approaching the Gaussian distribution by summing a finite number of values from a uniformly distributed sequence. The probability distribution of x is

$$P_N(x) = \begin{cases} \dfrac{\left[1 - (-1)^{N-x+1}\right] \; N!}{2^{N+1} \left[N-(x+N)/2\right]! \; \left[(x+N)/2\right]!} & \text{when } |x| \leq N \\ 0, & \text{when } |x| > N \end{cases} \tag{A-2}$$

When N = ∞, Equation A-2 becomes a Gaussion distribution (Reference 11). In the practical realization, according to Neuvo and Ku, N is chosen to be between 100 and 4000.

The properties and characteristics of PRBS's have been the subject of other investigations also[9, 12, 13, 14]. A matter of interest in many of these investigations has been the hypothesis that the distribution is Gaussian. Using 32,000 output samples, Neuvo and Ku (Reference 11) determined that a chi-squared goodness-of-fit test for $N = 1000$ indicated there was no reason to reject the hypothesis that the distribution is Gaussian at the five percent level of significance. Jordan and Wood (Reference 13) concluded that N should satisfy the condition $144 \leq N << 2^n$, in order to produce an approximately binomially distributed pseudorandom variable by assuming $N > n$ bits of a maximal-length sequence produced by an n-bit shift register.

When a narrow-band filter is excited by a PSK SS signal involving a PRBS, it performs in a way that somewhat resembles Neuvo and Ku's noise source shown in Figure A-1. The narrow-band filter maintains a running sum of the last N consecutive bits, or chips, of the incoming SS signal, where $N = B_c/B_r$; i.e., N is the number of chips received during the impulse response time $(1/B_r)$ of the filter. If the filter output were sampled at intervals of $1/B_r$, the sample values would have essentially the same properties as the samples of x in Neuvo and Ku's noise source.

When $B_c/B_r >> 1$, the output of the narrow-band filter is nearly proportional to the sum of the (+1, -1) bits received during the impulse

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[12]Davies, A.C., "Properties of Waveforms Obtained by Nonrecursive Digital Filtering of Pseudorandom Binary Sequences," *IEEE Trans. on Computers*, Vol. C-20, No. 3, March 1971, p. 270-281.

[13]Jordon, H. and D. Wood, "On the Distribution of Sums of Successive Bits of Shift-Resgister Sequences, *IEEE Trans. on Computers*, April 1973, p. 400-408.

[14]Cumming, I.G., "Autocorrelation Function and Spectrum, Pseudorandom Binary Sequence," *Proc. IEE*, Vol. 114, No. 9, September 1967, pp. 1360 1362.

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PROCEDURES FOR ANALYZING INTERFERENCE CAUSED BY SPREAD-SPECTRUM--ETC(U)
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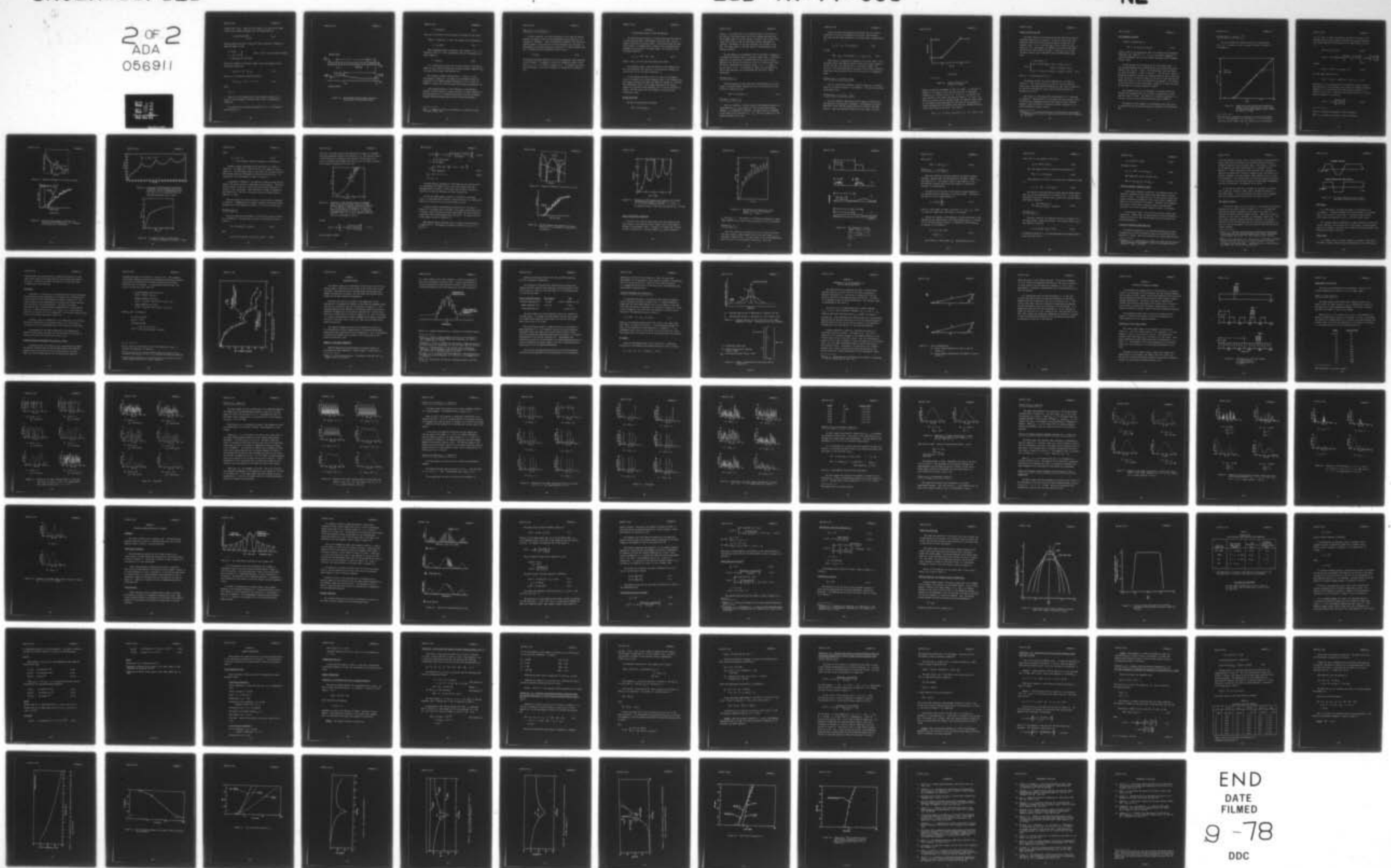

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response time $(1/B_r)$. When the input signal is on tune ($\Delta f=0$) sample values of the output envelopes $\upsilon_o\{t_N\}$ are approximated by

$$\upsilon_o\{t_N\} \approx (B_r/B_c) \sum_{j=k-N}^{k} a_j \qquad \text{(A-3)}$$

where t_N denotes the time at which the filter response is sampled, as shown in Figure A-2 and

$$\left.\begin{aligned} N &= B_c/B_r \\ k &= (t_N - \Delta)/\tau \end{aligned}\right\} \quad \text{Note: N and k are the nearest integers}$$

Δ = time delay in the filter

The term a_j depends on the input signal, which for purposes of this discussion is given by:

$$\upsilon_{in}\{t\} = a\{t\} \sin \omega_c t \qquad \text{(A-4)}$$

where $a\{t\}$ is the binary modulating function

$$a\{t\} = a_k, \quad k\tau \leq t < (k+1)\tau \qquad \text{(A-5)}$$

where

$$a_k = \pm 1$$

When $\Delta f \neq 0$, the output power of the response waveform is reduced by the off-frequency rejection (OFR), which is explained in APPENDIX B.

The variance of the response waveform for $\Delta f = 0$ is obtained by using Equation A-1.

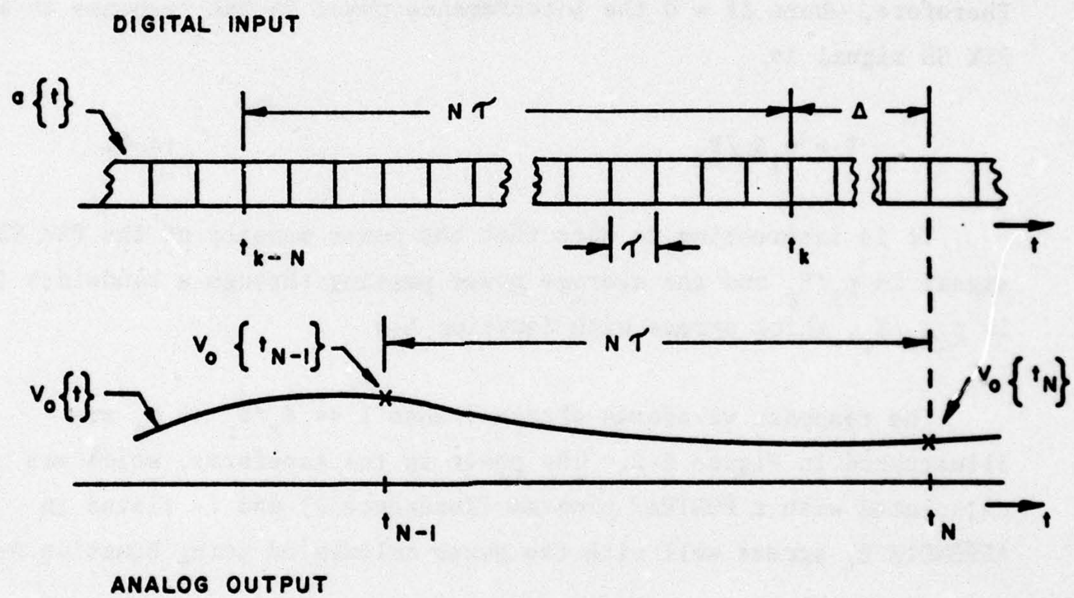

Figure A-2. Relationship between digital input and analog output of narrow band filter.

$$\sigma^2 = Np_i(\tau B_r)^2 \tag{A-6}$$

where p_i is the power of the SS signal at the input of the filter.

Since $N = B_c/B_r$ and $\tau = 1/B_c$, the variance can be expressed as

$$\sigma^2 = p_i \; B_r/B_c \tag{A-7}$$

The interference power is defined as the variance, i.e., $\sigma^2 = 1$. Therefore, where $\Delta f = 0$ the interference power in the response to a PSK SS signal is

$$I = p_i B_r/B_c \tag{A-8}$$

It is interesting to note that the power density of the PSK SS signal is p_i/B_c and the average power passing through a bandwidth (B_r) is $p_i B_r/B_c$, which agrees with Equation A-8.

The response waveforms obtained when $1 << B_t/B_r << N_c$ are illustrated in Figure E-2. The power in the waveforms, which was calculated with a FORTRAN program (Reference 2) and is listed in APPENDIX E, agrees well with the power calculated using Equation A-8.

A more elegant analysis of the response to a pseudorandom sequence could be obtained by using the results of Lee's analysis of the autocorrelation of a waveform consisting of the sum of impulse responses having starting times that are Poisson distributed.[15]

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[15] Lee, Y., *Statistical Theory of Communication*, John Wiley & Sons, New York, 1960 p. 336.

<u>When $B_t/B_r >> N_c$ and $B_i/B_r << 1$</u>

For this condition, the filter bandwidth is less than the spacing between the spectral lines of the pseudorandom code of length N_c, as explained in APPENDIX F. When the information bandwidth (B_i) of the SS spectrum signal is less than the receiver bandwidth (B_r), the power in the response is essentially equal to the power in a single spectral line that is modulated by the information signal, that is

$$I \approx p_i/N_c \tag{A-9}$$

The response waveform resembles that of the information signal contained in the SS signal. An example, in which the information signal is constant (i.e., $B_i = 0$) is shown in Figure E-3-e. For purposes of setting the INR threshold, this kind of response is treated here as being CW-like.

APPENDIX B

THE REJECTION FACTORS IN THE INR EQUATION

The INR equation in Section 2 includes three rejection factors: on-tune rejection (OTR), off-frequency rejection (OFR), and the signal-discontinuity rejection (SDR). These terms relate the power of the output of a stagger-tuned, bandpass filter (Butterworth or Chebyshev) to the power of the input signal, as indicated in the following expression:

$$P_o = P_i - OTR - OFR - SDR \tag{B-1}$$

where P_i and P_o are the input and output power (dBm).

This appendix gives a short explanation of the formulas used for calculating OTR, OFR, and SDR. A more detailed treatment of these terms will be included in Reference 2.

In the explanations that follow, the input to the bandpass filter is assumed to be a direct-sequence signal with a bandwidth B_t. The signals involving PSK, QPSK, and OQPSK are assumed to be derived from trapezoidal pulses, with lengths depending on the coding sequence and rise/fall times depending on the time for the phase to change. The PFM signals are also assumed to be derived from trapezoidal pulses.

ON-TUNE REJECTION

The OTR is defined here as follows:

$$OTR \equiv 10 \log (p_i/p_o) \tag{B-2}$$

where p_i is the power (mW) of the incoming signal at the filter input and p_o is the output power (mW) of the response waveform when the incoming signal is on-tune and has a duration that is many times longer than $(1/B_r)$ so that no significant gating transients are involved. The effects of any such transients are included in the SDR. The output power is the mean square of the amplitude of the response waveform.

The OTR depends on the bandwidth ratio B_t/B_r which, in practice, can range from extremely small to extremely large values. Therefore, a capability is required for calculating OTR for any value of B_t/B_r. Equations that describe the behavior of OTR as a function B_t/B_r were derived by considering the intervals $(B_t/B_r << 1)$, $(1 << B_t/B_r << N_c)$ and $(B_t/B_r >> N_c)$. As indicated in Figure B-1, the behavior of OTR in these intervals can be described with relatively simple expressions.

OTR when $B_t/B_r << 1$

When the passband is much wider than the bandwidth of the input signal, the signal passes through the filter with essentially no distortion or rejection so that

$$OTR = 0, \text{ for } B_t/B_r << 1 \qquad \text{(B-3)}$$

OTR when $1 << B_t/B_r << N_c$

With this condition, the mean square of the response waveform is approximated by $p_o = s_d B_r$, where s_d is the spectral power density (mW/MHz). As explained in APPENDIX F, the power density (in dBm/MHz) at the center of the spectrum is $P_i + S_p - 10 \log B_t$, where S_p is the spectrum peaking factor (dB).

Using the above relationships and assuming that the receiver selectivity is unity for frequencies within the 3 dB bandwidth (B_r) and zero for other frequencies, the output power is approximated by

$$P_o = P_i + S_p - 10 \log (B_t/B_r) \qquad \text{(B-4)}$$

so that

$$OTR = -S_p + 10 \log (B_t/B_r), \; 1 << B_t/B_r << N_c \qquad \text{(B-5)}$$

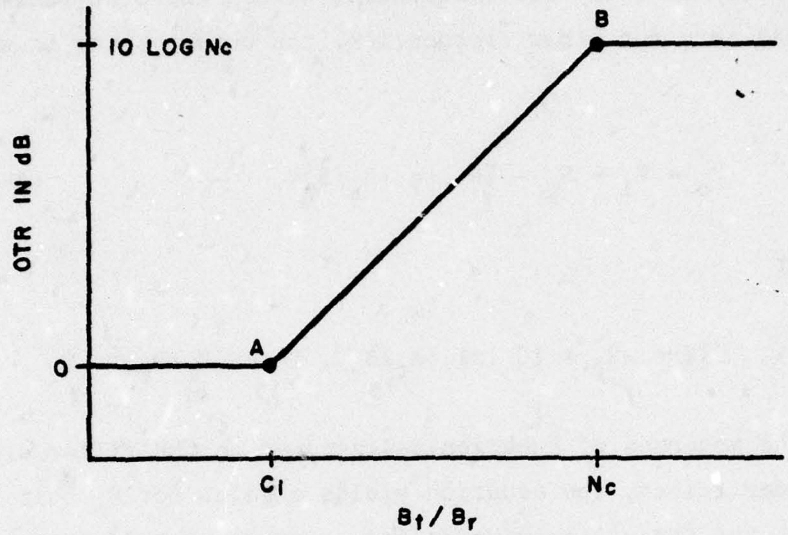
The accuracy of Equation B-4 depends on the filter order. For a 2nd-order filter, the equation yields a value for P_o that is about 3 dB low, and for a 4th-order filter, a value that is about 1 dB in error. The error diminishes as the order of the filter increases, which causes the selectivity curve to take on more of a rectangular shape.

OTR When $B_t/B_r >> N_c$ and $B_i \geq B_t/N_c$

For this conditon the spectrum of the SS signal is a continuous function, as shown in Figure F-2 in APPENDIX F. The OTR is calculated using Equation B-5.

OTR When $B_t/B_r >> N_c$ and $B_i << B_t/N_c$

For this condition, spectrum of the SS signal is treated as a line spectrum, as discussed in APPENDIX F. Furthermore, the filter bandwidth is less than the spacing between the lines of the power spectrum. When the passband overlaps one of the lines, the power



(Log Scale)

$c_1 \approx 1$ to 2

Figure B-1. General behavior of OTR as B_t/B_r is varied.

output is the power represented by that line (dBm). As indicated in APPENDIX F, the center line of the power spectrum is suppressed (Reference 14), having a value of P_i - 20 log N_c + S_p; whereas, the lines adjacent to the center line have the value P_i - 10 log N_c + S_p. When B_r is smaller than the spacing between the spectral lines, which is the condtion being addressed here, the power in the response is usually determined by the adjacent lines that pass through the skirts of the filter selectivity curve. Thus, OTR is

$$OTR \gtrless - S_p + 10 \log N_c, \text{ for } B_t/B_r \gg N_c, \quad B_i \ll B_t/N_c \qquad \text{(B-6)}$$

General Equation for OTR

The above formulas for calculating OTR were derived for specific bandwidth conditions, as indicated. It should be noted that these bandwidth conditions do not provide contiguous coverage over the range of $B_t/B_r = 0$ to ∞; there are gaps in the vicinity of $B_t/B_r = 1$ and $B_t/B_r = N_c$. Fortunately, the formulas yield sufficiently good results even in the vicinity of $B_t/B_r = 1$ and $B_t/B_r = N_c$ so that the above formulas can be combined to form the OTR equation given below.

$$\text{OTR} = \begin{cases} 0, \text{ for } B_t/B_r < c_1 \\ -S_p + 10 \log (B_t/B_r), \text{ for } c_1 < B_t/B_r \leq N_c \text{ or} \\ \qquad B_t/B_r > N_c \text{ and } B_i \geq B_t/N_c \\ -S_p + 10 \log N_c, \text{ for } B_t/B_r > N_c \text{ and } B_i < B_t/B_r \end{cases} \quad \text{(B-7)}$$

where $c_1 = \text{antilog } (S_p/10) \approx 1 \text{ to } 2$

The behavior of OTR, as given by Equation B-7, is shown in Figure B-1. The values of B_t/B_r at points (A) and (B) where the breaks occur, were determined by equating the right sides of Equations B-3 and B-5 and solving for B_t/B_r, and also doing the same with Equations B-5 and B-6.

The accuracy in calculating OTR using Equation B-7 is indicated in Figure B-2. The data points in the figure are very accurate values of OTR that were determined using the DSR (digital-signal response) program[16] which calculates the response waveforms and power in responses produced by digital signals.

[16]Newhouse, P. D., *Closed-Form Equations for Calculating the Responses of Bandpass Filters to Digital Signals*, ECAC-TN-78-009, ECAC, Annapolis, MD, July 1978.

OFF-FREQUENCY REJECTION

The OFR is defined here by

$$\mathrm{OFR} = 10 \log (p_o \{0\}/p_o\{\Delta f\}) \qquad \text{(B-8)}$$

where $p_o \{\Delta f\}$ is the output power (mW) as a function of the frequency separation (Δf), where $\Delta f = f_t - f_r$, f_r is the center frequency of the filter passband, and f_t is the carrier frequency of the incoming digital signal.

The OFR depends on the spectrum of the interfering signal. It can be approximated using the bounds $\tilde{E}_t\{\Delta f\}$ on the emission spectrum, where $\tilde{E}_t \{\Delta f\} = \tilde{F}_t\{\Delta f\} - R_t\{\Delta f\}$. The OFR formulas using the bounds are given in TABLES 4, 6, and 8 in Section 2. The OFR can be approximated more accurately, however, by using the spectral density functions $F_t\{\Delta f\}$ and $F_\delta\{\Delta f\}$ so that the effects of the nulls in the spectrum are taken into account, as will be explained in the appendix. The effects of the nulls can be of major importance, especially in analyzing interference caused by frequency-hopping SS signals.

The procedures given in this appendix for calculating OFR are based on the assumption that the SS signal is generated by a power amplifier that is linear. The effects of a nonlinear amplifier (i.e., one that is driven into saturation) are discussed in APPENDIX F.

The behavior of OFR depends on the bandwidth ratio B_t/B_r which is evident from the following discussion of equations for calculating OFR.

OFR when $B_t/B_r < 1$ and $B_r\delta < 1$[a]

For this condition the response consists of impulses when $|\Delta f| >> 0$. An example of such a response is shown in Figure E-4 in APPENDIX E.

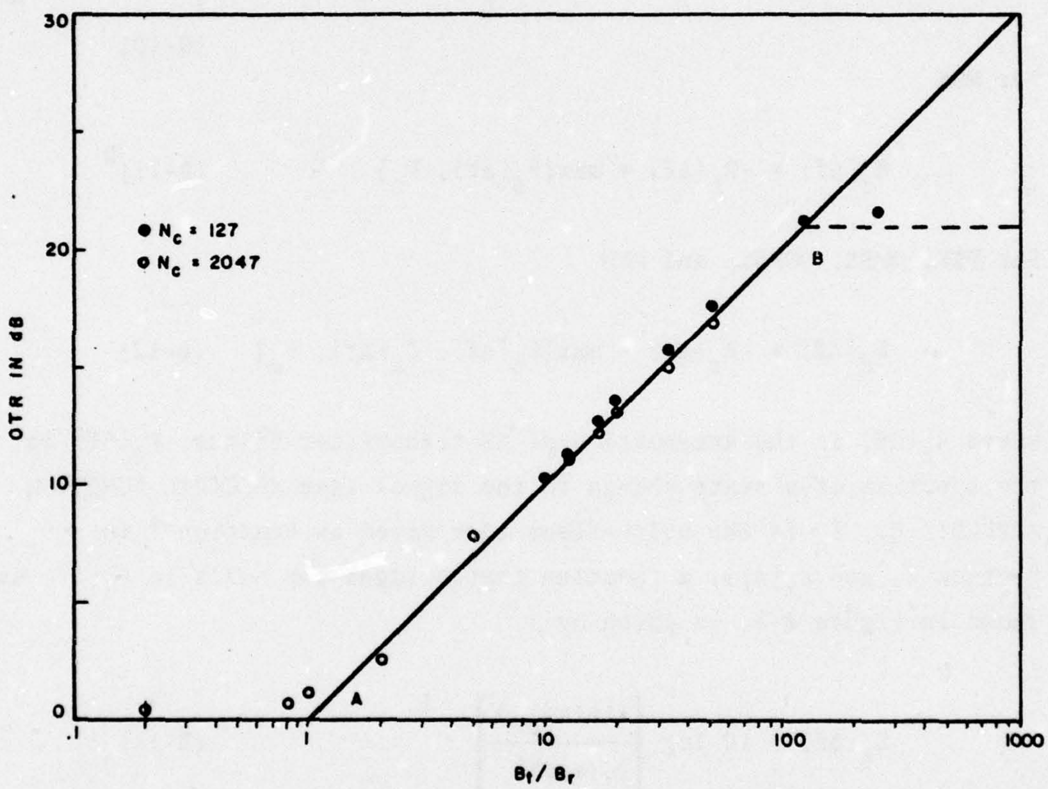


Figure B-2. Comparison of OTR obtained with Equation B-7 (graph) and DSR Program (points); PSK signal, 4th-order Butterworth bandpass filter; in most SS systems $N_c > 8000$.

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[a] Practically all situations are expected to satisfy the condition $B_r\delta < 1$. The procedures to be used when $B_r\delta > 1$ are not within the scope of this report; they are, however, given in Reference 2.

The OFR, which is used in calculating the INR of the peaks of the impulses caused by state changes (excluding the impulses caused by the leading edge and trailing edges of the signal burst) is given by

$$\mathrm{OFR} = \mathrm{Max}\ [0,\ R_I\{\Delta f\}] \qquad \text{(B-9)}^a$$

$$R_I\{\Delta f\} = -20 \log \left\{ \mathrm{antilog} \left[\frac{E_\delta\{\Delta f\} + 20 \log B_r}{20} \right] + \mathrm{antilog} \left[\frac{-R_r\{\Delta f\}}{20} \right] \right\} \qquad \text{(B-10)}$$

For MSK

$$E_\delta\{\Delta f\} = -R_t\{\Delta f\} + \max[F_\delta\{\Delta f\},\ F_n] \qquad \text{(B-11)}^b$$

For PSK, QPSK, OQPSK, and PFM

$$E_\delta\{\Delta f\} = -R_t\{\Delta f\} + \max[F_\delta\{\Delta f\},\ Z_\delta\{\Delta f\},\ F_n] \qquad \text{(B-12)}$$

where $R_t\{\Delta f\}$ is the attenuation of SS transmitter filter, $F_\delta\{\Delta f\}$ is the spectrum of a state change in the signal (see SPECTRAL FUNCTION, APPENDIX F), F_n is the noise-floor term given by Equation 8 in Section 2, and $Z_\delta\{\Delta f\}$, a function that bridges the nulls in $F_\delta\{\Delta f\}$ as shown in Figure B-3, is given by

$$Z_\delta\{\Delta f\} = 10 \log \left[\frac{\sin(\pi B_{r\varepsilon} \delta)}{2\delta (\pi \Delta f)^2} \right]^2 \qquad \text{(B-13)}$$

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[a]Min [x, y] denotes the minimum of the two arguments.

[b]Max [x, y] denotes the maximum of the two arguments.

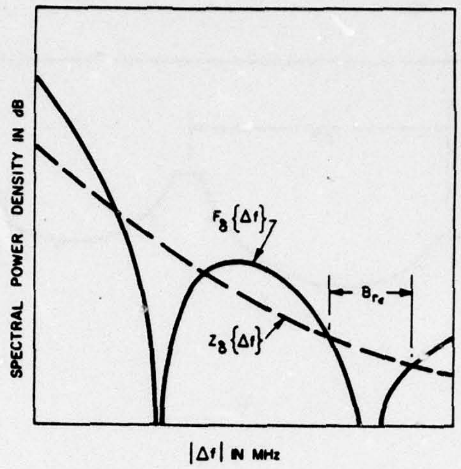

Figure B-3. Graphical treatment of the nulls in $F_\delta\{\Delta f\}$.

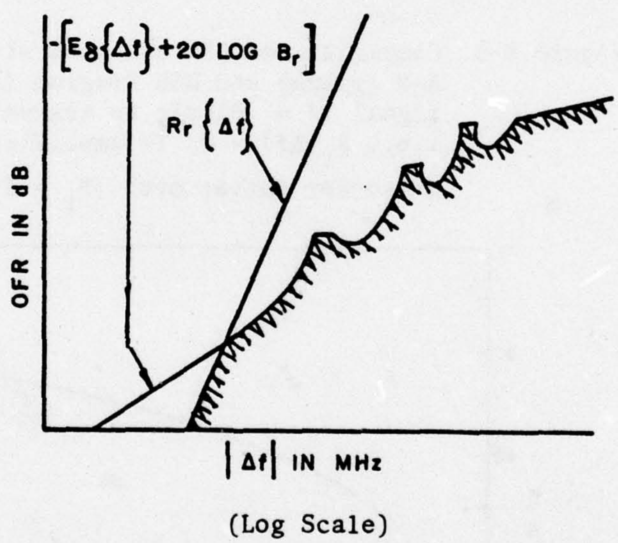

Figure B-4. The OFR (shaded segments) obtained with Equation B-9 (used when $B_t/B_r < 1$). Referred to as Option No. 3 in Section 2.

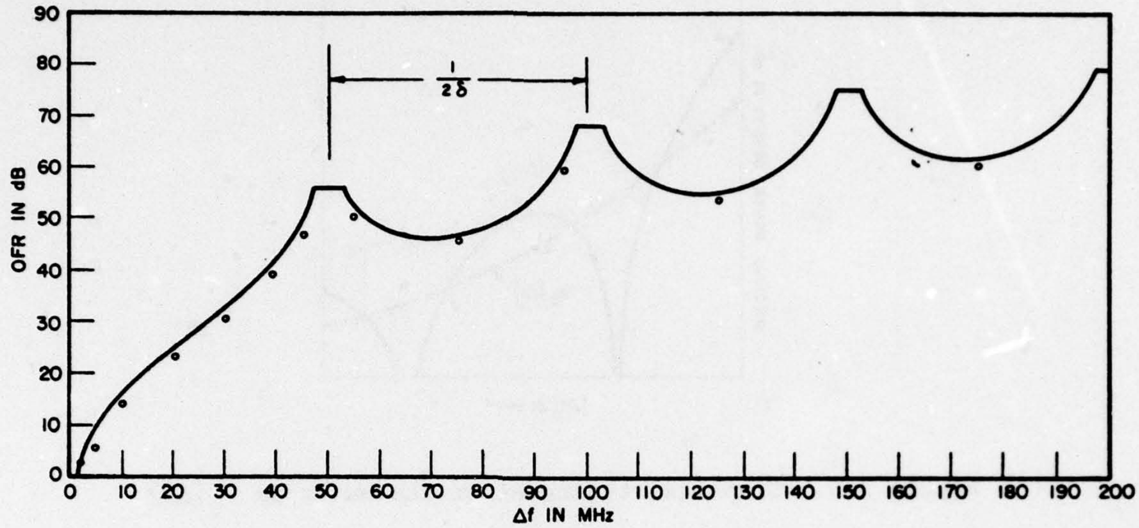

Figure B-5. Comparison of OFR obtained with Equation B-9 (graph) and DSR Program (Points); PSK signal (δ = .01μs); no transmitter filter, i.e., $R_t\{\Delta f\} = 0$; IF amplifier filter is 4th-order Butterworth (B_r = 5 MHz).

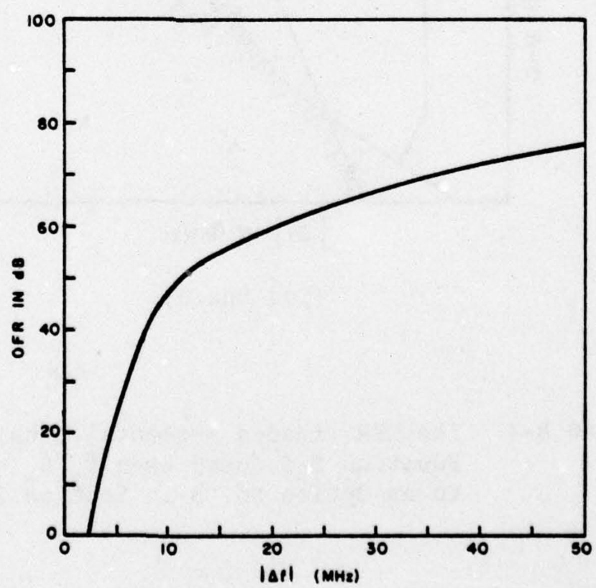

Figure B-6. An example of OFR for an MSK signal (τ = 1μs, no transmitter filter, B_r = 5 MHz).

where

$$B_{r\varepsilon} = B_r + 2\varepsilon \qquad \text{(B-14)}$$

ε = the frequency stability tolerance of the receiver.

A general graphic representation of Equaiton B-9 is shown in Figure B-4. An example of a plot of OFR for a PSK signal is shown in Figure B-5. The data points shown in the figure are accurate values of OFR obtained using the DSR program, which was cited previously. An example plot for an MSK signal is shown in Figure B-5.

As explained in APPENDIX F, the spectral bounds are often used instead of the actual spectra in representing the SS signals. The bounds are much easier to use and in many cases the attendent reduction in accuracy is not significant. When the bounds are used in conjunction with Equation B-9, the OFR is calculated using the formulas given in TABLES 4, 6, and 8.

When the bounds are used the accuracy is not as good as indicated in Figure B-7. Depending on the accuracy required, one chooses either Equation B-10 or the formulas in TABLE 4, 6, or 8 to calculate $R_I\{\Delta f\}$.

OFR When $B_t/B_r \geq 1$

With this condition the response is noise-like as shown in Figure E-2 in APPENDIX E. The OFR, which is used in calculating the INR, is given as

$$\text{OFR} = \text{Min}\ [R_r\{\Delta f\},\ -E_t\{\Delta f\}] \qquad \text{(B-15)}$$

where

$$E_t\{\Delta f\} = \text{Max}\ [F_t\{\Delta f\},\ Z_t\{\Delta f\},\ F_n]\ -R_t\{\Delta f\} \qquad \text{(B-16)}$$

and $F_t\{\Delta f\}$, the power density spectrum of the SS signal, is obtained using the appropriate formula from APPENDIX F; F_n, the spectral density of the noise floor is obtained using Equation 8 in Section 2; and $Z_t\{\Delta f\}$, a function that bridges the nulls in $F_t\{\Delta f\}$ as shown in Figure B-8 is given by Equations B-17a and B-17b.

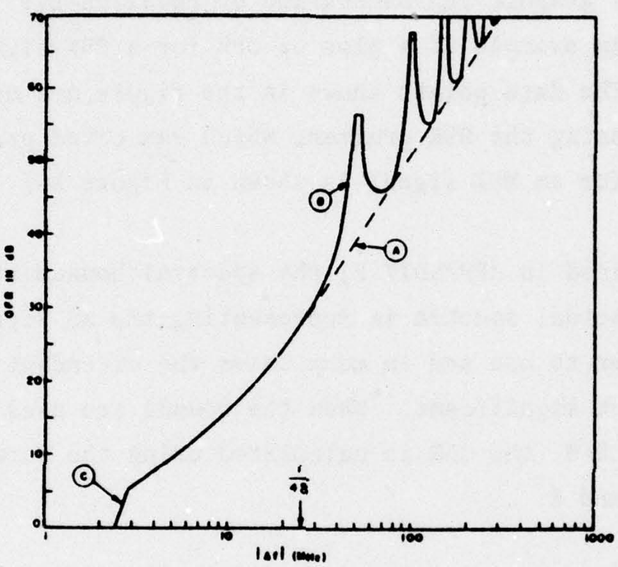

Figure B-7. Comparison of OFR obtained using the procedure given at the bottom of page 15 for a wideband receiver (Curve A, which is referred to as Option No. 2 in Section 2) and Equation B-9 (Curve B, which is referred to as Option No. 3); PSK with no transmitter filter (δ = .01 μs), 4th-order IF amplifier filter (B_r = 5 MHz); segment C of curves A and B is determined by $R_r\{\Delta f\}$.

For MSK

$$Z_t\{\Delta f\} = \text{Min}\left[0,\ 10\log\left|\frac{\sin(\pi B_{r\varepsilon}\tau)}{1+(4\tau\Delta f)^2}\right|^2\right] \qquad \text{(B-17a)}$$

For PSK, QPSK, or OQPSK

$$Z_t\{\Delta f\} = \left[\text{Min} \quad 0,\ 10 \log \left\{ \frac{\sin(\pi B_{r\varepsilon} \tau A/2)\ \sin(\pi \delta B)}{2A\tau\delta(\pi\Delta f)^2} \right\}^2 \right] \tag{B-17b}$$

$$A = \begin{cases} 1 \text{ for PSK or QPSK} \\ 2 \text{ for OQPSK} \end{cases}$$

$$B = \begin{cases} B_{r\varepsilon}, \text{ when } (\Delta f_d - \frac{B_{r\varepsilon}}{2}) < |\Delta f| < \Delta f_d + \frac{B_{r\varepsilon}}{2} \\ 2\Delta f, \text{ otherwise} \end{cases} \tag{B-17c}$$

$$\Delta f_d = d/2\delta,\ d = 1, 2, 3, \ldots \tag{B-18a}$$

$$B_{r\varepsilon} = B_r + 2\varepsilon$$

The power density spectrum of a PFM signal has very shallow nulls. As a consequence the bounds, $\tilde{E}_t\{\Delta f\}$, that are obtained using the procedure given in Figure 12, are always used in place of $E_t\{\Delta f\}$ in Equation B-15 when the offending SS signal involves PFM.

For MSK and OQPSK signals, Equation B-15 should be used when $B_t/B_r > 1/2$ and Equation B-9 should be used when $B_t/B_r \leq 1/2$ because the width of the lobes of $F_t\{\Delta f\}$ for these signals is $B_t/2$.

A general graphic representation of Equation B-15 is shown in Figure B-9. An example plot of OFR for a PSK signal using Equation B-15 is shown in Figure B-10 which includes data points that were accurately calculated using the DSR program cited previously. An example plot for an MSK singal is shown in Figure B-11.

When spectral bounds are used, $\tilde{E}_t\{\Delta f\}$ is used in place of $E_t\{\Delta f_2\}$ in Equation B-15. The formulas are given in TABLES 4, 6, and 8 in Section 2.

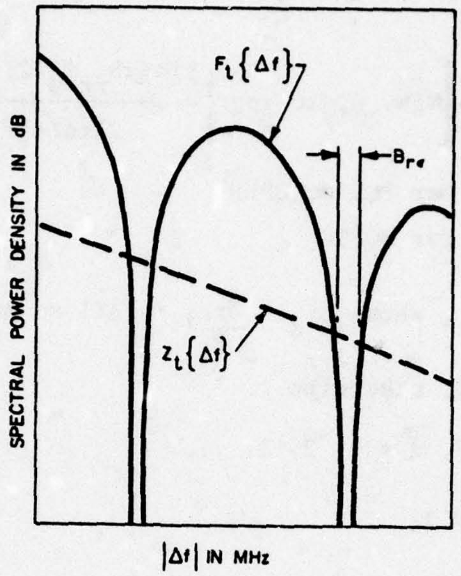

Figure B-8, Graphical treatment of the nulls in $F_t\{\Delta f\}$.

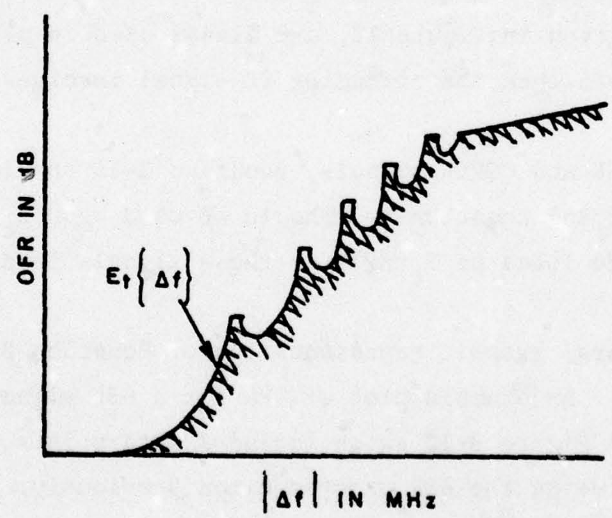

Figure B-9. The OFR obtained with Equation B-15 (used when $B_t/B_r \geq 1$), referred to as Option No. 3 in Section 2.

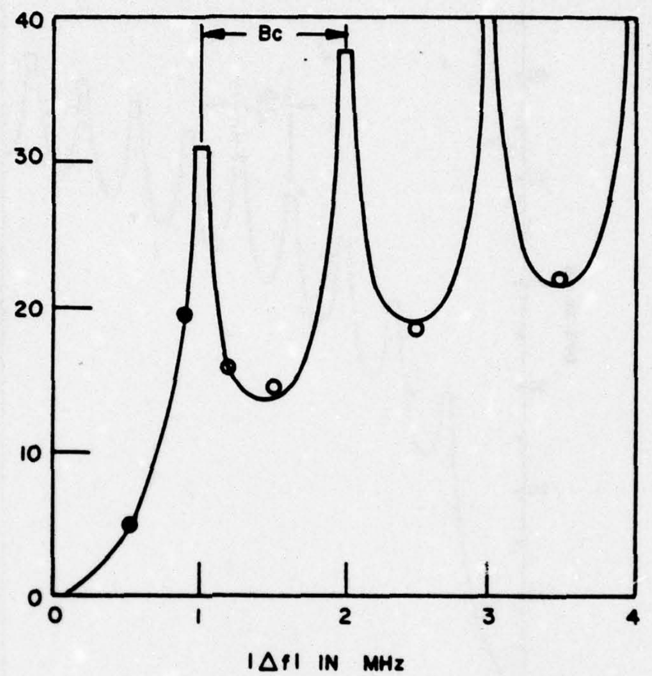

Figure B-10. Comparison of OFR obtained with Equation B-15 (Graph) and DSR program (points); PSK signal (τ = 1 μs, δ = .01 μs); no transmitter filter, i.e., $R_t\{\Delta f\} = 0$; IF amplifier filter is 4th-order Butterworth (B_r = 0.5 MHz).

SIGNAL-DISCONTINUITY REJECTION

The SDR is the rejection that results when the duration of the input signal is less than the nominal length ($1/B_r$) of the impulse response of the bandpass filter. Consider a SS signal that is gated as shown in Figure B-12. The gate length is τ_g and the spacing is

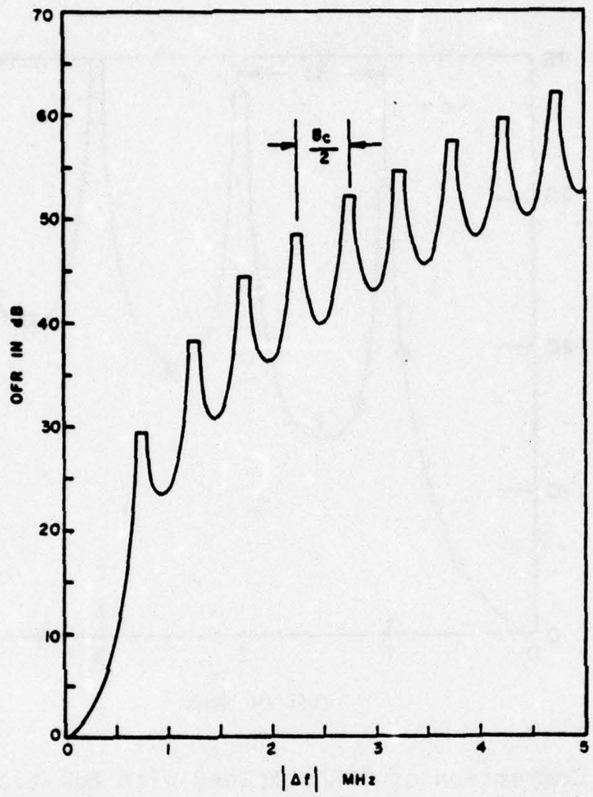

Figure B-11. OFR obtained with Equation B-15 MSK signal (τ = 1μs); no transmitter filter; B_r = .1 MHz.

T_g, and $B_t/B_r >> 1$. The nature of the responses produced by a gated signal depends on the values of $B_r\tau_g$ and B_rT_g, as will be explained.

SDR When $B_r\tau_g >> 1$.

With this condition the response consists of noise-like bursts. Each burst length is approximately equal to the gate length τ_g, as indicated in Figure B-12-a. The mean square of the waveform amplitude within the burst is not affected by the gating. Thus, the

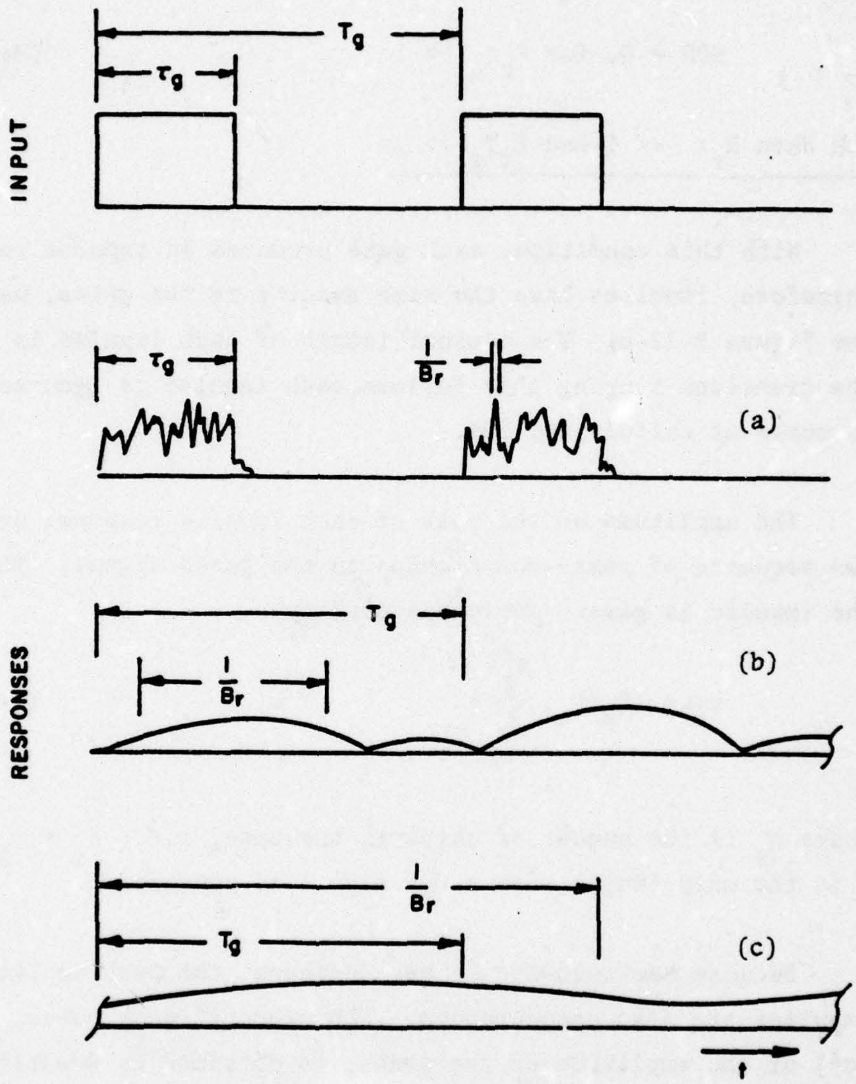

Figure B-12. The responses to a gated signal. $B_t/B_r >> 1$ in all cases. (a) $B_r\tau_g > 1$, (b) $B_r{}_g < 1$ and $B_rT_g > 1$, (c) $B_rT_g < 1$.

SDR in dB is

$$\text{SDR} = 0, \text{ for } B_r \tau_g >> 1 \tag{B-20}$$

<u>SDR When $B_r \tau << 1$ and $B_r T_g >> 1$</u>

With this condition, each gate produces an impulse response. Therefore, impulses have the same spacing as the gates, namely, T_g. See Figure B-12-b. The nominal length of each impulse is $1/B_r$. The transient ringing that follows each impulse is ignored for purposes of calculating SDR.

The amplitude of the peak of each impulse response depends on the sequence of phase-coded chips in the gated signal. The peak of the impulse is given (approximately) by

$$\hat{\upsilon}_o = (B_r/B_c) \sum_{j=1}^{N_g} a_j \tag{B-21}$$

where N_g is the number of chips in the gate, i.e., $N_g = \tau_g/\tau$, where τ is the chip length when a PSK signal is considered.

Because the sequence is pseudorandom, the peak amplitudes of the impulses are also pseudorandom. The expected peak power, or variance (σ^2) of the amplitude of the peaks, is obtained by substituting N_g in Equation A-6 of APPENDIX A.

$$\begin{aligned} \sigma^2 &= (\tau_g/\tau)\, p_i\, (\tau B_r)^2 \\ &= p_i B_r^{\,2}\, \tau_g\, \tau \end{aligned} \tag{B-22}$$

The variance is power output, p_o. Substituting p_o for σ^2, and B_t for $1/\tau$ into Equation (B-22) gives

$$p_o = p_i\ (B_r/B_t)\ (B_r\tau_g) \tag{B-23}$$

For a PSK signal the OTR is obtained using Equation B-5

$$\text{OTR} = 0 + 10 \log (B_t/B_r) \tag{B-24}$$

Using Equations B-23 and 24, the output power, expressed in dBm, is

$$P_o = P_i - \text{OTR} - 10 \log (B_r\tau_g)^{-1} \tag{B-25}$$

The rejection resulting from the signal duration being shorter than $1/B_r$ is the logarithmic term in Equation B-25. If the signal duration is equal to or greater than τ_g there is no rejection due to this effect; therefore,

$$\text{SDR} = \begin{cases} 0, \text{ when } B_r\tau_g \geq 1 \\ 10 \log (B_r\tau_g)^{-1}, \text{ otherwise} \end{cases} \tag{B-26}$$

SDR When $B_rT_g << 1$

With this condition, the impulses overlap as in Figure B-12-c. The output power is derived as follows: the expected energy (e_o) in each impulse is given by

$$e_o = p_i\ (B_r/B_t)\ (B_r\tau_g)\ (1/B_r) \tag{B-27}$$

Dividing the energy by T_g, the spacing between the incoming bursts, gives the output power

$$p_o = p_i (B_r/B_t)(\tau_g/T_g) \tag{B-28}$$

The power in dBm is

$$P_o = P_i - OTR - 10 \log (T_g/\tau_g) \tag{B-29}$$

The logarithmic term is the SDR, thus,

$$SDR = 10 \log (T_g/\tau_g), \text{ for } B_r T_g < 1 \tag{B-30}$$

EFFECT OF ROUNDED CORNERS ON PULSES

If the pulses should be trapezoidal with rounded corners, the spectrum, and hence the OFR, would be the same except for large values of $|\Delta f|$[17]. At large values of $|\Delta f|$ the emission spectra are usually corrupted by transmitter noise, which tends to negate the effects of the rounding of the corners. Therefore, the assumption that pulses in many types of digital signals are trapezoidal, especially in the case of binary coding as in PSK, is reasonable.

In MSK digital signals, the pulses are sinuoidal rather than trapezoidal. Studies that are being conducted at ECAC (Reference 2) indicate that even for MSK, the OFR expressions derived above are good approximations.

EFFECTS OF NONLINEAR POWER AMPLIFIER

The spectral equations in the foregoing explanations are good representations of the SS transmitter emission, provided the power amplifier is linear. When the transmitter has a power amplifier that

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[17]Newhouse, P. D., "Simple Realistic Models for Radar Emission Spectra", *IEEE Southeastern Electromagnetic Compatibility Sympsium Record*, October 27, 1969, p. 222-236.

limits the signals, however, such as the traveling-wave-tube amplifiers used in many satellite systems, the resulting emission spectrum is different from that of an unlimited signal. The effects of limiting on the spectrum have been investigated by S.A. Rhodes[18] and H.R. Mathwick.[19] These investigations were concerned with a transmitter emission that passes first through a bandpass filter that reduces the spectral sidelobes, and then through a final amplifier that is driven into saturation. The limiting tends to restore the spectral sidelobes to their original levels and thus negates, to some degree (depending on the type of modulation), the benefits of the filter.

In the case of binary PSK, the effect of limiting is relatively easy to analyze, whereas for QPSK, OQPSK, and MSK the analysis is more complicated. A brief discussion of the subject and a suggested empirical procedure for treating limited SS signals follows.

PSK (Binary) Signals

A binary PSK signal can be described as a carrier that is amplitude-modulated by the function shown in Figure B-13. It is assumed here that when the phase is changed from one state to the other, the amplitude-modulating function changes linearly. When such a signal is limited, its rise time is shortened, as indicated in Figure B-13. The pulse shape, however, remains trapezoidal; therefore, the spectral functions for the limited signal can be calculated using Equation F-9 and F-10, with the parameter δ given the value of the rise time of the limited waveform.

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[18]Rhodes, S.A., "Effects of Hardlimiting on Bandlimited Transmissions with Conventional and Offset QPSK Modulation, *National Telecommunications Record,* NTC Record 1972, IEEE, Houston, TX pg. 204-207.

[19]Mathwich, H.R., Balcewicz, J.F., and Hecht, M., "The Effect of Tandem Band and Amplitude Limiting on the E_b/N_o Performance of Minimum (Frequency) Shift Keying (MSK)," *IEEE Transaction on Comminications,* Vol. COM-22, No. 10, Oct 1974 pg 1525-1540.

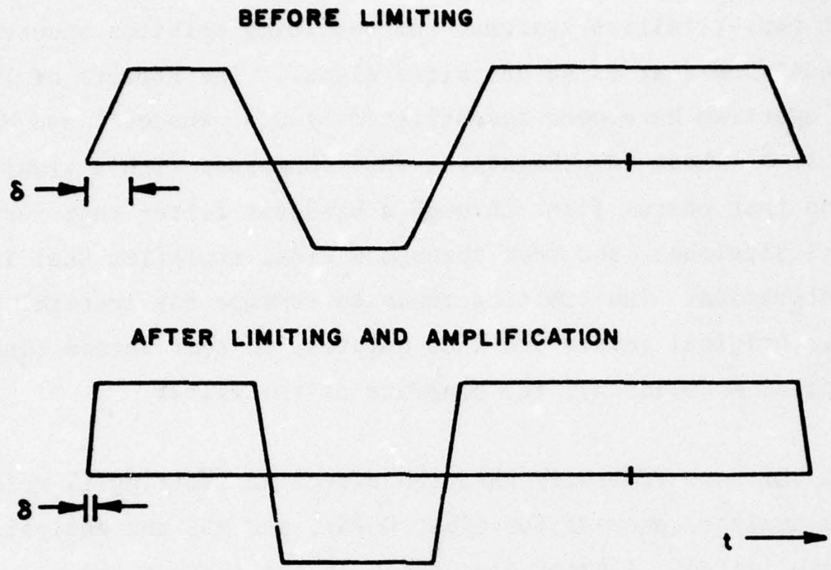

Figure B-13. The modulating function of a binary PSK signal, before and after limiting.

QPSK Signal

In a QPSK signal, the phase may change by 180 degrees or by ± 90 degrees. A 180-degree change causes the amplitude to go through zero, as in the case of binary PSK. A 90-degree change, however, merely causes the amplitude to fall to .707 of the peak value.

S/A. Rhodes' analysis (Reference 18), which involved using the Fast Fourier Transform, indicated that with a QPSK signal, limiting negated entirely the effects a 4th-order filter ahead of the power amplifier and shortened the depth of the nulls.

OQPSK Signal

In an OQPSK signal, the phase changes by 90 degrees; there are no 180-degree changes. Rhodes' analysis (Reference 18) of a OQPSK signal indicated that the limiting partially negated the effects of a 4th-order filter ahead of the power amplifier and shortened the depth of the nulls. The result was about the same as if a 2nd-order filter had been used with no limiting.

MSK Signal

The effects of limiting on a filtered MSK signal were investigated both theoretically and experimentally by Mathwich, et al (Reference 19). Typical results of their measurements are shown in Figure B-14. The limited signal used in measurements was obtained with a power amplifier that was driven 10 dB into saturation. The filter (3rd-order Butterworth) used in that particular measurement had a 3-dB bandwidth that was 1.1 times the chip rate; i.e., $B_F/B_C = 1.1$, a value which we believe to be typical for an SS transmitter. The limiting raised the sidelobes of the filtered spectrum by about 5 dB.

In Rhodes' analysis (Reference 18) of an MSK signal, the limiting raised the sidelobes of the spectrum by about 10 dB. The filter in his analysis had a 3-dB bandwidth equal to the chip rate of MSK signal.

An examination of the results obtained by both Mathwich and Rhodes with MSK signals indicated that the limiting had the effect of reducing the order of the filter by a factor of two.

SUGGESTED EMPIRICAL PROCEDURE FOR LIMITED SS SIGNALS

As mentioned above, the results of the investigations by Rhodes and Mathwich indicated that for PSK and QPSK signals, the limiting negates entirely the effects of any filtering ahead of the power amplifier; and for OQPSK and MSK, the limiting has the effect of reducing the order of the filter by a factor of two. This suggests the following empirical procedure for determining the emission spectrum $E_t\{\Delta f\}$ and its bounds $\tilde{E}_t\{\Delta f\}$ if neither measured data nor better analytical data is available. In calculating these functions, use $R_{ts}\{\Delta f\}$ in place of $R_t\{\Delta f\}$, that is:[a]

1. In the procedure given on page 15
   $\tilde{E}_t\{\Delta f\} = \tilde{F}_t\{\Delta f\} - R_{ts}\{\Delta f\}$
2. Replace Equation B-16 with
   $E_t\{\Delta f\} = -R_{ts}\{\Delta f\} + \text{Max}\ [F_t\{\Delta f\}, Z_t\{\Delta f\}, F_n]$
3. Replace Equation B-12 with
   $E_\delta\{\Delta f\} = -R_{ts}\{\Delta f\} + \text{Max}\ [F_\delta\{\Delta f\}, Z_\delta\{\Delta f\}, F_n]$

where $R_{ts}\{\Delta f\}$ is obtained by

For PSK and QPSK[b,c]

$R_{ts}\{\Delta f\} = 0\text{ dB}$

For OQPSK and MSK

$$R_{ts}\{\Delta f\} = \begin{cases} 0, & \text{for } |\Delta f| < B_F/2 \\ 10\ p\ \log|2\Delta f/B_F|, & \text{otherwise} \end{cases}$$

---

[a] $R_t\{\Delta f\}$, which is the insertion loss of the transmitter filter, is obtained with Equation 4 in Section 2.

[b] If the rise time of the limited waveform should be known or can be estimated, use Equation F-9 or F-10 to calculate the spectral functions.

[c] A more accurate procedure is to calculate and plot the spectrum of the limited signal using the Fast Fourier Transform.

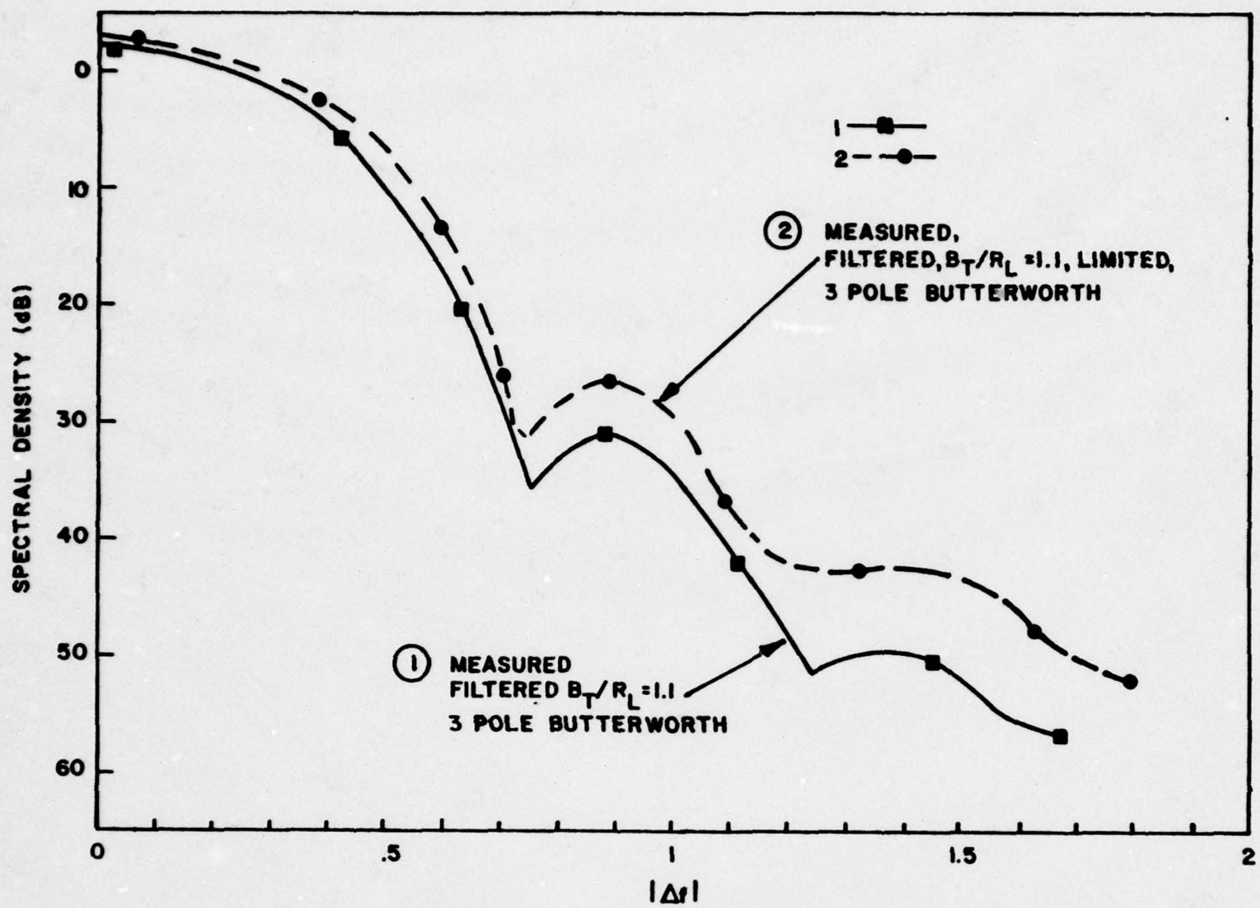

Figure B-14. Measured spectra showing the effects of limiting on a filtered MSK signal (Reference 32).

APPENDIX C

TRANSMITTER NOISE

The emitted signal of a transmitter includes noise as well as the desired signal. This noise can be traced to the source noise of the oscillator, the additive noise of the amplifiers and microwave components following the oscillator, and the random and periodic perturbations of the power supplies and modulator.

The noise has little or no effect on the upper part of the spectrum but it determines the behavior of the far-out sidebands, which fall off at a relatively slow rate as indicated in Figure C-1. The calculated bounds on the spectrum of the desired signal can be used to represent the upper part of the emission spectrum but a "noise floor," which has a constant level as shown in Figures 10, 11, and 12 of Section 2, is used as the bound on the lower sidebands of the emission spectrum. The level of the noise floor is set at F_n (dB) below the peak of the normalized spectrum.

This appendix summarizes some of the information available in the literature on the noise characteristics of microwave power-amplifying devices that might be used in SS transmitters. Also, an explanation is given of the formula used in Section 2 for calculating F_n, the relative noise-floor level.

SUMMARY OF PUBLISHED INFORMATION

Numerous papers have been published on the noise spectra of oscillators and power amplifiers. Edson's paper[20] on the noise in

[20] Edson, W., "Noise in Oscillators," *Procedures of the IRE,* Vol. 48, August, 1960, p. 1454-1466.

Oscillators seems to be a basic reference. Kuvas[21] and Kurokawa[22] have also investigated oscillator noise. The noise performance of klystron amplifiers has been measured by Klaus[23], and the noise of solid-state amplifiers have been measured by Willing[24] and Omori[25].

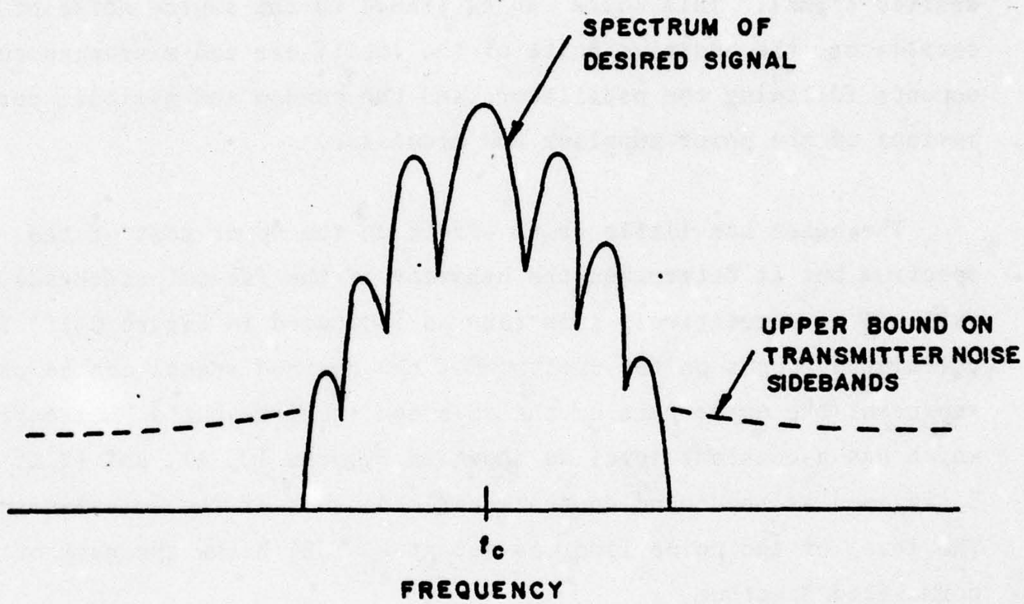

Figure C-1. General characteristics of spectrum of transmitted signal.

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[21]Kuvas, R., "Noise in Single-Frequency Oscillators and Amplifiers," *IEEE Transaction on Microwave Theory and Techniques*, Vol. MTT-21, No. 3, March 1973, p. 127-134.

[22]Kurokawa, K., "Noise in Synchronized Oscillators," *IEEE Transaction on Microwave Theory and Techniques*, Vol. MTT-16, April 1968, p. 234-240.

[23]Klaus, S., "The Measurement of Near-Carrier Noise in Microwave Amplifiers," *IEEE Transaction on Microwave Theory and Techniques*, Vol. MTT-16, No. 9, September 1968, p. 761-766.

[24]Willing, H., "A Two-Stage IMPATT-Diode Amplifer," *IEEE Transaction on Microwave Theory and Techniques*, Vol. MTT-21, No. 11, November 1973 p. 707-716.

[25]Omori, M., "Gunn Diodes and Sources," *Microwave Journal*, June 1974 p. 57-62.

ECAC has investigated noise in HF, VHF, and UHF transmitters but not in microwave transmitters.

The references cited above have reported the following noise figures and noise spectral power densities, which are given in terms of the ratio N/C, when N is the noise power in a 1-Hz bandwidth and C is the carrier power.

Type of Amplifier Device	Noise Figure	N/C
Klystron (Reference 22)	28 (dB)	-140 to -150(dB in 1 Hz)
IMPATT (Reference 23)	30 to 55	-150
GUNN (Reference 24)	-----	-160 to -170

The noise figure for a power amplifier varies as a function of the power input. For example, Willing reports that the noise figure of an IMPATT power amplifier varies from 42 to 52 dB as the input power varies from 14 to 23 dBm.

The noise figure concept is reported to be not as convenient in power amplifiers as it is in receivers because most power amplifiers (especially those that employ solid-state devices) are not linear. The noise produces both amplitude perturbations and phase, or frequency, perturbations of the carrier (Reference 20). Measurements and theoretical analysis of the performance of a power amplifying device involve two separate and distinct spectra, one for the amplitude perturbations and one for the phase perturbations.

Just how well the measured data given in the various references cited above would represent the noise performance of SS transmitters operating in the field is not known yet. ECAC will make some measurements on SS transmitters, which should be helpful in determining the level of the noise floor. Until such data become available, it is suggested that the level of the noise floor be determined using the procedure explained below.

SUGGESTED PROCEDURE FOR CALCULATING F_n

As explained previously, the spectral power densities reported in the literature are given in terms of N/C. It seems reasonable to assume that the noise power density (mW in 1-Hz) remains at the same level when the carrier is modulated, as indicated in Figure C-2. Going on that assumption, the noise floor with respect to the peak of the spectrum of the modulated carrier is given by

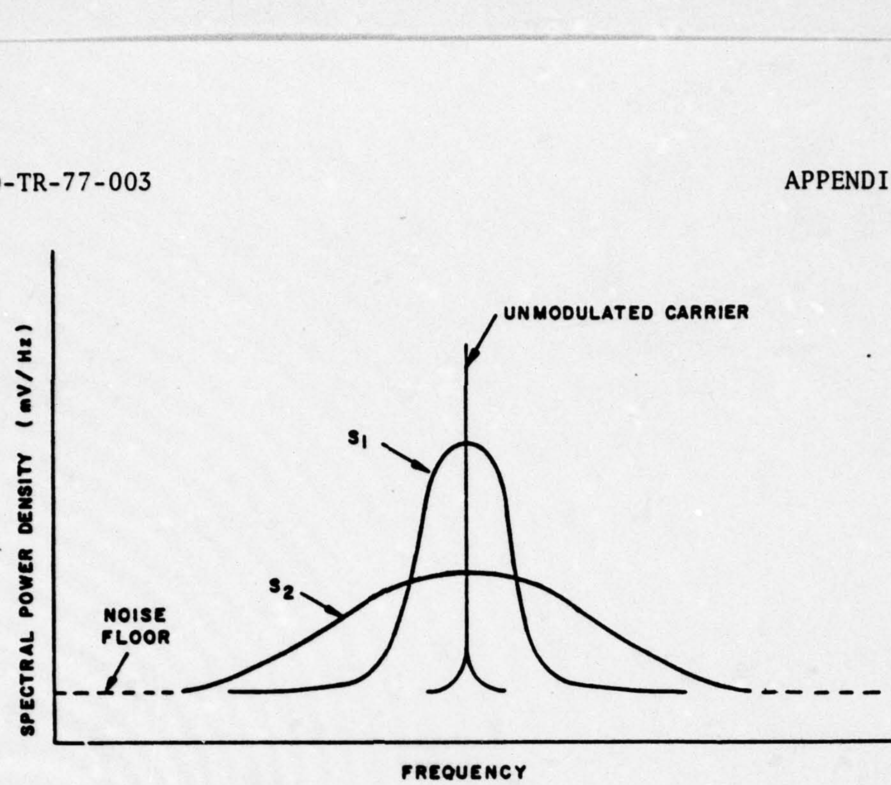
$$F_n = (N/C) + 60 - (S_p - 10 \log B_t) \qquad \text{(C-1)}$$

where S_p, the spectrum peaking factor (dB), and B_t, the signal bandwidth (MHz), are defined in Section 2. The relationship between the terms in this equation are shown graphically in Figure C-3. It is recommended that a value of -140 (dB in 1 Hz) be used for N/C, unless some measured data on the particular SS transmitter should be available.

An Example

Using the recommended value for N/C, Equation C-1 yields the following for a PSK modulated SS signal having a bit rate of 10 Mb/s;

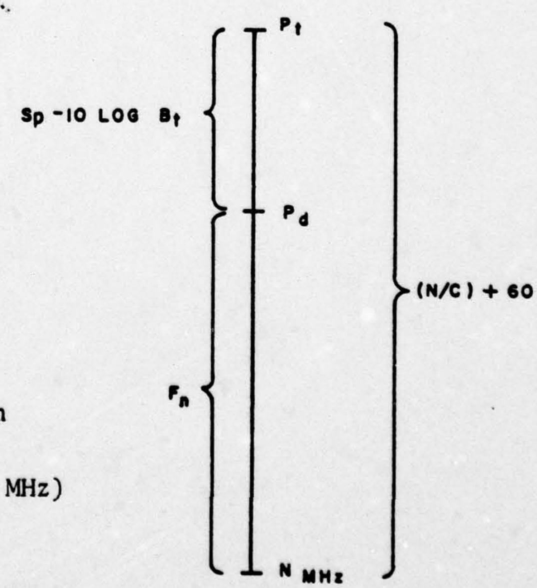
$$F_n = -140 + 60 - (0 - 10 \log 10) = - 70 \text{ dB}.$$



S_1 = Spectrum when carrier is modulated at a moderate bit rate

S_2 - Spectrum when carrier is modulated at a very high bit rate

Figure C-2. Expected behavior of the noise floor as the signal bandwidth is varied. (transmitter power held constant).



P_t = Transmitter power (dB)

P_d = Peak of power density spectrum (dBm in 1 MHz)

N_{MHz} = Noise power density (dBm in 1 MHz)

Figure C-3. Graphic representation of the terms used in Equation C-1.

APPENDIX D

DETECTABILITY OF SS INTERFERENCE IN AM, FM, AND PM RECEIVERS

The traditional way of analyzing the effect of band-limited Gaussian noise on a carrier involves the vector configuration depicted in Figure D-1-a. The carrier is denoted by V_c and the noise V_n, which has a Rayleigh amplitude distribution. The phase (ϕ) is assumed to have a uniform distribution. The resultant signal is represented by the vector V_r.

The vector V_n can be represented by an in-phase component ($V_{I_n} = V_n \cos \phi$) and a quadrature component ($V_{Q_n} = V_n \sin \phi$). V_{I_n} causes an amplitude perturbation and V_{Q_n}, a phase (or frequency) perturbation. Edson (Reference 20) has determined the autocorrelation function and the spectrum associated with each of these perturbations. Mullen[26] points out that the AM receiver responds to the amplitude perturbations and an FM receiver, to the phase perturbations.

A similar approach is useful in visualizing the effects of interference from an SS signal. See Figure D-1-b. The carrier frequency of the transient response is the midband frequency of the desired signal at the IF-amplifier output. When the interference is caused by a PSK signal the phase (ϕ) could be restricted to two discrete values, or with a QPSK interference signal, could be restricted to four discrete values. In practice, ϕ is expected to vary somewhat randomly over a range of values due to phase fluctuations of the desired signal and to phase fluctuations in the interference signal

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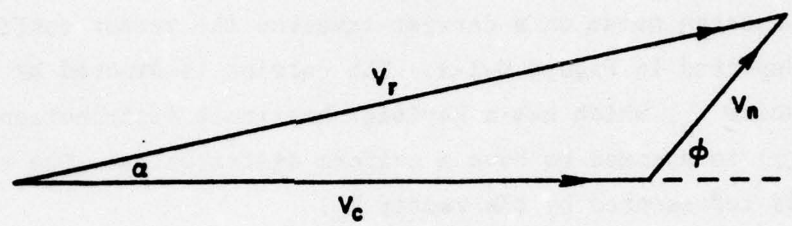
[26] Mullen, J., "Background Noise in Nonlinear Oscillators," *Proceedings of the IRE,* August 1960, p. 1467-1473.

A



B

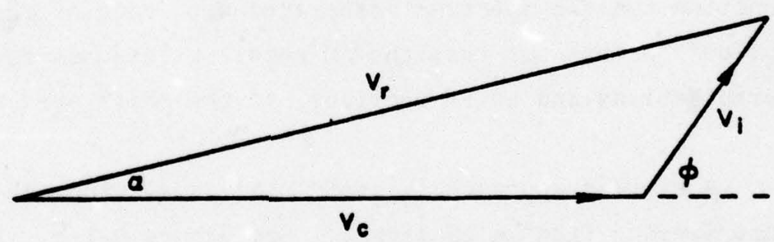


Figure D-1. Vector representation.

(a) Vector diagram representing noise V_n and the carrier V_c.

(b) Vector diagram representing interference V_i and the carrier V_c.

which take place in the transmission path. The in-phase component, $V_i \cos \phi$, would cause amplitude perturbations that would be detected in an AM receiver, and the quadrature component, $V_i \sin \phi$, would cause phase, or frequency, perturbations that would be detected in an FM or PM receiver.

It's conceivable that with PSK interference, $\phi = 0$ and 180°, in which case there would be no phase perturbations and the interference would not be detected in an FM or PM receiver; or $\phi = \pm 90°$, in which case there would be essentially no amplitude perturbations and the interference would not be detected in an AM receiver. It is assumed here that even with PSK interference, ϕ is not confined to values of 0 and 180° nor ± 90°, except for very short periods of time. Thus, both in-phase and quadrature components are assumed to be present so that the perturbations produced by the interference would always be detected by an AM, FM, or PM receiver.

APPENDIX E

EXAMPLES OF RESPONSE WAVEFORMS

The waveforms and average powers of the responses of a bandpass filter to an assortment of direct-sequence (DS) and frequency-hopping direct-sequence (FH/DS) signals involving PSK modulation were calculated and the envelopes plotted using the Digital Signal Response computer program (Reference 16). The results were used to theoretically validate the procedures pertaining to PSK signals given in Section 2 and to obtain an insight into the general characteristics of the responses produced by digital signals involving pseudorandom sequences.

This appendix presents some of the plots obtained with PSK signals to illustrate the effects of the filter bandwidth and signal parameters on the response waveforms.

Description of the Input Signals

The various input signals are configured as illustrated in Figure E-1. All of the signals have a 1-volt amplitude, a 1-μs chip length (τ), and a .01-μs rise/fall time (δ). The pseudorandom sequences have code lengths (N_c) of either 127 or 2047 chips. These inputs represent DS signals from unfiltered SS transmitters; i.e., $R_t\{\Delta f\} = 0$. The bandpass filter, which represents the IF-amplifier filter of a victim receiver, is a fourth-order Butterworth in all cases.

Although actual SS signals usually involve much shorter chip lengths than 1 μs and longer code lengths than 127 or 2047 chips, the results of these calculations and plots help provide the desired insight into the response waveform characteristics.

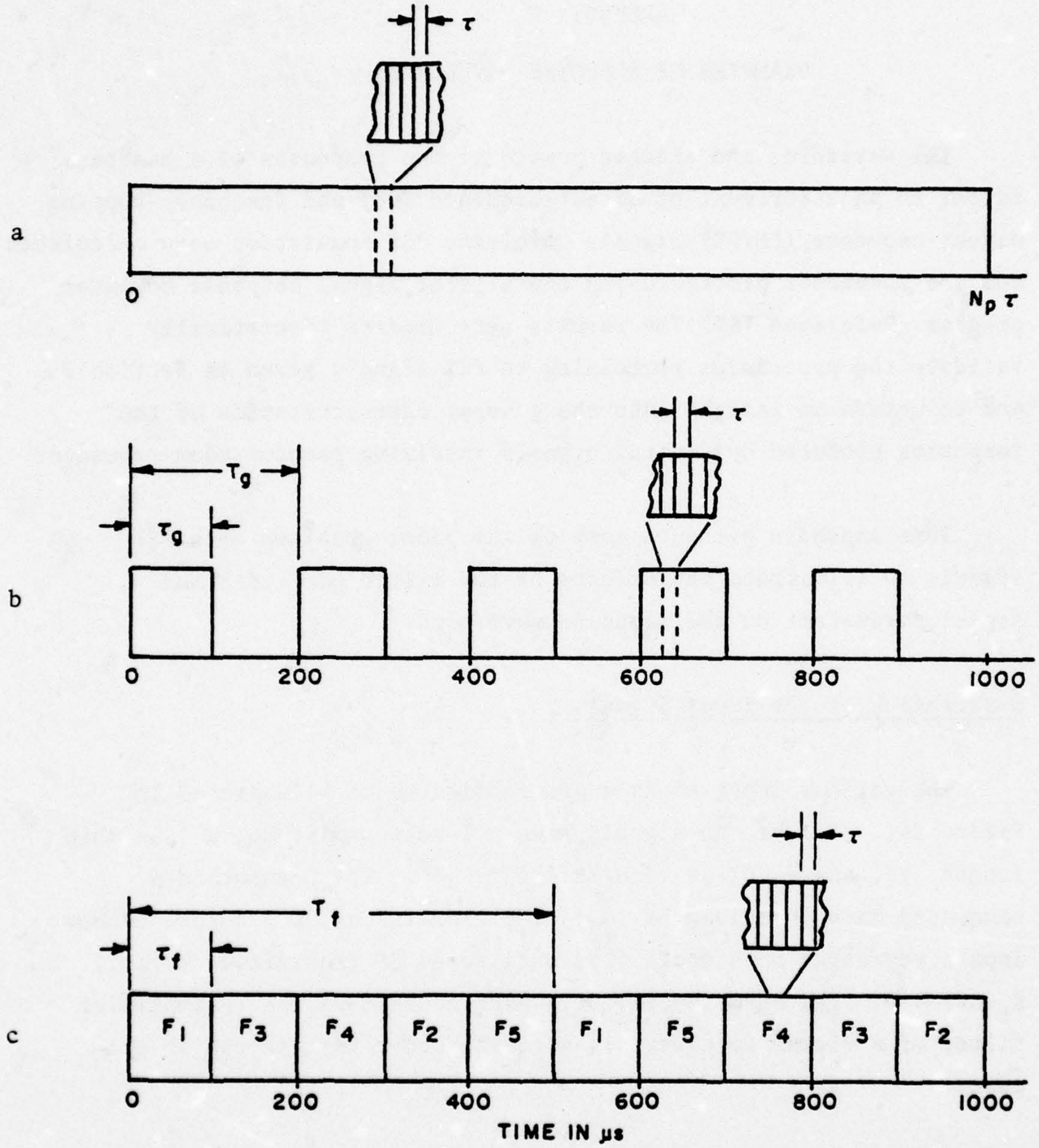


Figure E-1. Configuration of the input signals.
(a) single-gated DS
(b) multiple-gated DS,
(c) FH/DS

Explanations of the Plots

The plots are the envelopes of the responses. The time is in μs and the amplitude is relative to a 1-volt input signal.

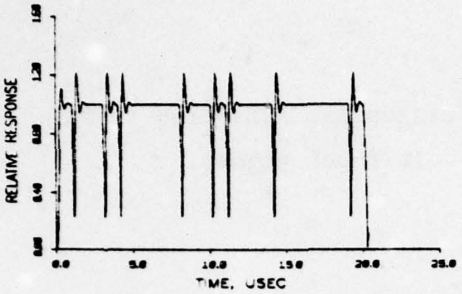
Effects of B_t/B_r Figure E-2

The input signals that produce these responses consist of N_p chips so that the signal duration is $N_p\tau$ as indicated in Figure E-2a. The length (N_c) of the pseudorandom sequence, 2047 chips, is larger than the total number of chips in the signals.

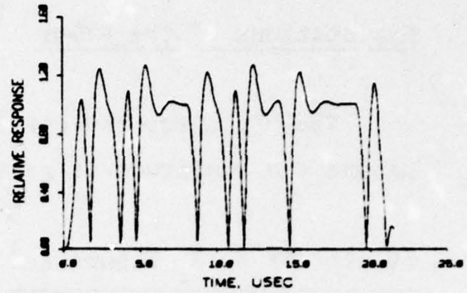
When $B_t/B_r \leq 1$, as in Figures E-2a, b, and c, the responses are essentially undistorted; each chip is resolved. As B_t/B_r is increased, the chips are no longer resolved, as in Figure E-2d, and eventually the waveform takes on a noise-like appearance, as in Figures E-2e through l. The average power for each of these waveforms is:

Figure	Average Power[a]
E-2-(a)	.95
(b)	.79
(c)	.74
(d)	.46
(e)	.17
(f)	.076
(g)	.058
(h)	.049
(i)	.031
(j)	.020
(k)	.0061
(l)	.0018

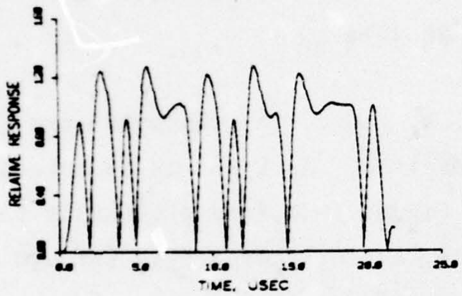
[a]The input power is one unit of power.



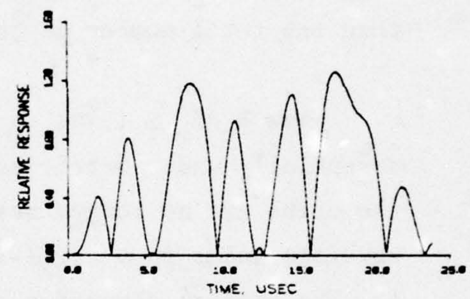
(a) $B_t/B_r = .2$
$N_p = 20$ pulses



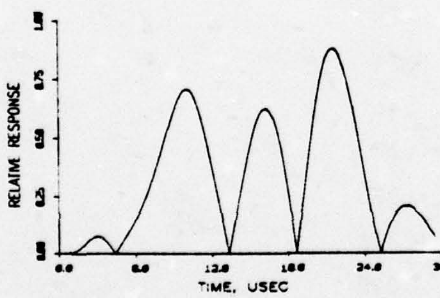
(b) $B_t/B_r = .8$
$N_p = 20$ pulses



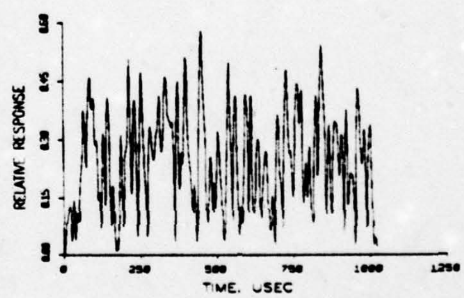
(c) $B_t/B_r = 1.0$
$N_p = 20$ pulses



(d) $B_t/B_r = 2$
$N_p = 20$ pulses

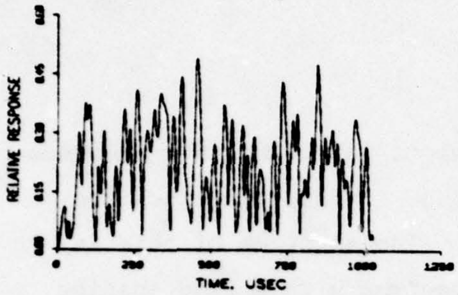


(e) $B_t/B_r = 5$
$N_p = 20$ pulses

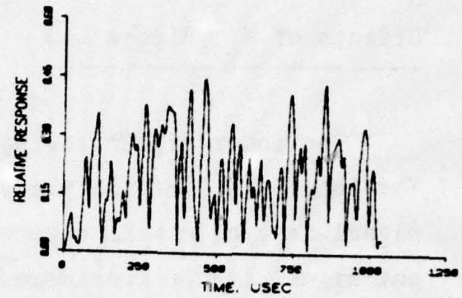


(f) $B_t/B_r = 12.5$
$N_p = 1000$ pulses

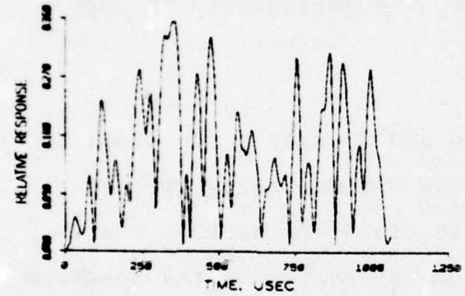
Figure E-2. Response to DS signal showing effects of B_t/B_r when code length is long ($N_c = 2047$, code length longer than signal duration, $\Delta f = 0$, $\tau = 1$ μs).



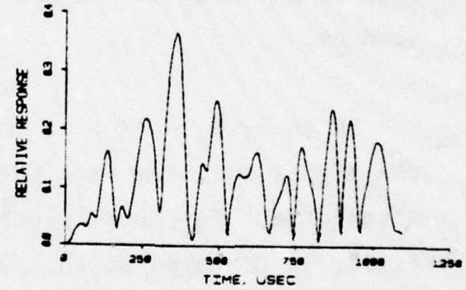
(g) $B_t/B_r = 17$
$N_p = 1000$ pulses



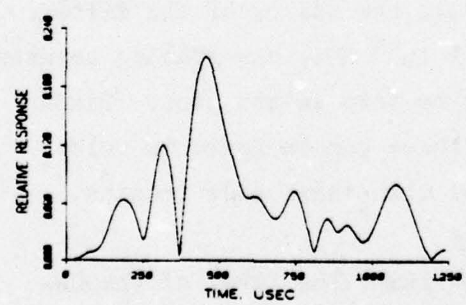
(h) $B_t/B_r = 20$
$N_p = 1000$ pulses



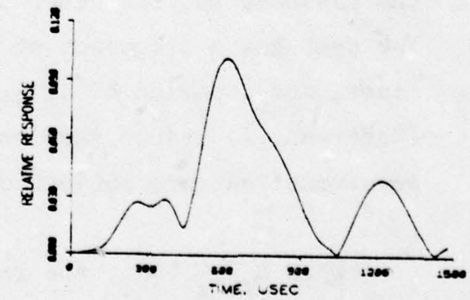
(i) $B_t/B_r = 33$
$N_p = 1000$ pulses



(j) $B_t/B_r = 50$
$N_p = 1000$ pulses



(k) $B_t/B_r = 125$
$N_p = 1000$ pulses



(l) $B_t/B_r = 250$
$N_p = 1000$ pulses

Figure E-2. (continued)

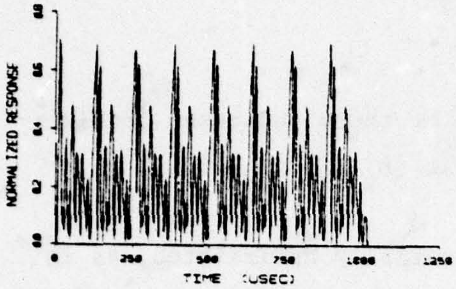
Effects of N_c, Figure E-3

The input signal for each of these plots is a 1000-chip sequence. The length (N_c) of the pseudorandom sequence is only 127, thus the signal is cyclic with a period of 127 μs. The spectrum of this input signal can be considered as a line spectrum with a line spacing of $B_c/N_c = 7.87 \times 10^{-3}$ MHz.

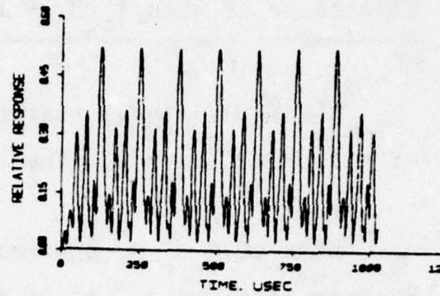
When $B_t/B_r \ll N_c$, as in Figures E-3a and b, the response is noise-like within each 127-μs interval; however, the periodicity is very evident.

When $B_t/B_r \to N_c$, as in Figures E-3c and d, only a few lines in the signal spectrum pass through the filter without being greatly attenuated. For these plots, Δf is set at .1575 MHz so that the filter is centered on the second line from the center of the spectrum rather than the center line which is suppressed as shown in Figure F-2a in APPENDIX F. The waveform in Figure E-3d is produced by three lines in the signal spectrum; one line passes through the center of the passband and the other two pass through the skirts of the filter. The beat has a frequency of $B_c/N_c = 7.9 \times 10^{-3}$ MHz, the spacing between lines, and a period of 127 μs, which can be seen in the plot. Dixon (Reference 1) states that beats such as these can be heard in voice receivers that are subjected to SS signal with short code lengths.

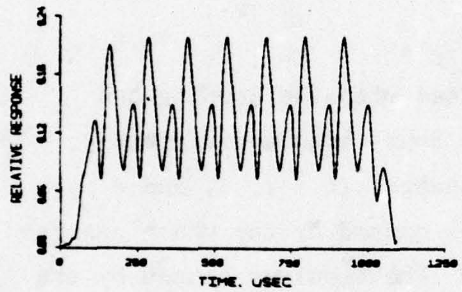
When $B_t/B_r > N_c$, the response is CW-like. The level of the CW-like response corresponds to the amplitude of the response at $t \approx 1000$ μs in Figure E-3e. If the input signal continued instead of being terminated at $t = 1000$ μs, the output would be of constant amplitude after the transients have diminished.



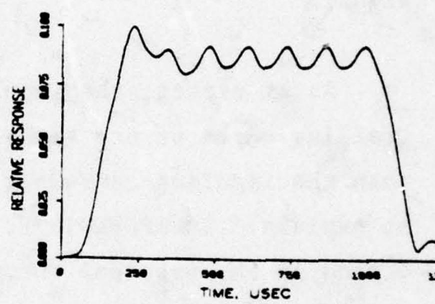
(a) $B_t/B_r = 10 < N_c$



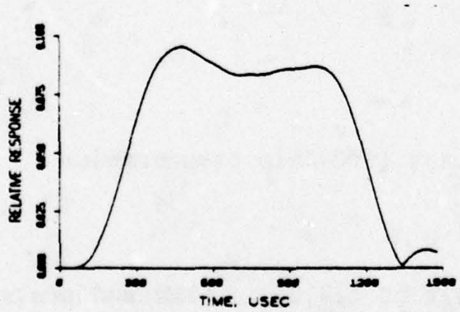
(b) $B_t/B_r = 17 < N_c$



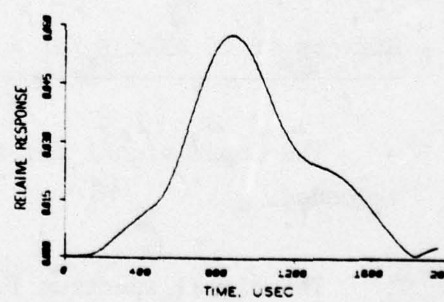
(c) $B_t/B_r = 50 < N_c$



(d) $B_t/B_r = 125 < N_c$



(e) $B_t/B_r = 250 > N_c$



(f) $B_t/B_r = 500 > N_c$

Figure E-3. Response to DS signal showing effects of B_t/B_r when code length is short (N_c = 127, code length = 127 μs, $\Delta f = 0$, τ = 1 μs, signal duration = 1000 μs).

Effects of Δf when $B_t/B_r < 1$, Figure E-4

The input signal that produces each of these responses consists of a 5-chip sequence. The bandwidth ratio (B_t/B_r) is 0.2.

When $\Delta f \leq B_r/2$, the response is essentially undistorted, as in Figures E-4a and b. As Δf is increased further, the response consists of an impulse each time the phase is changed, as in Figure E-4c through 1. Impulses are also produced by the leading and trailing edges of the signal.

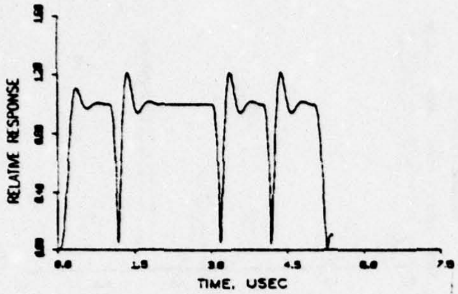
As Δf varies, the impulses associated with the leading and trailing edges of the signal ($t = 0$ and 5 μs) behave differently than the impulses caused by the phase changes ($t = 1$, 3, and 4 μs). As explained in APPENDIX F, the impulses caused by the phase changes depend on the spectral function $F_\delta\{\Delta f\}$. The impulses caused by the leading and trailing edges, which are of secondary importance when the signal consists of a large number of chips, can be calculated by substituting $\delta/2$ for δ in $F_\delta\{\Delta f\}$.

Effects of Δf When $B_t/B_r > 1$, Figure E-5

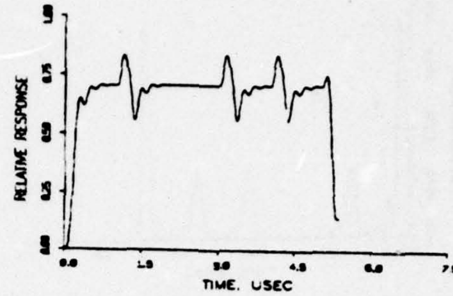
The input signal for these plots is a 1000-chip pseudorandom sequence.

The signal spectrum lobes have nulls at 1,2,3, ... MHz and peaks at 1.5, 2.5, 3.5, ... MHz. The bandwidth ratio (B_t/B_r) is 20.

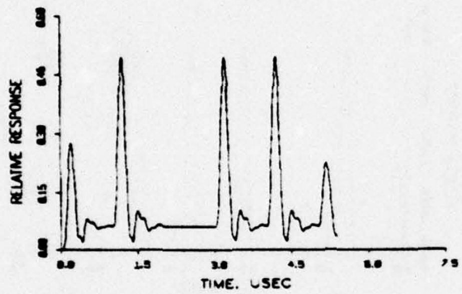
The average power for each of the noise-like responses is:



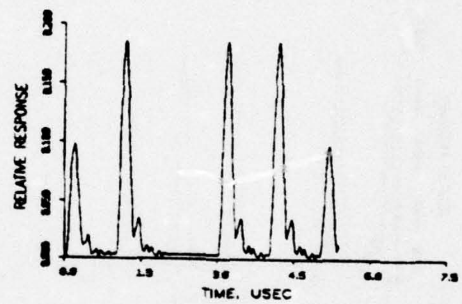
(a) $\Delta f/B_r = 0$



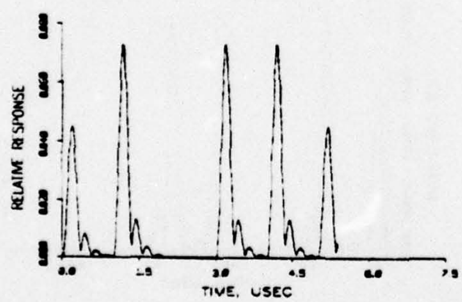
(b) $\Delta f/B_r = 1/2$



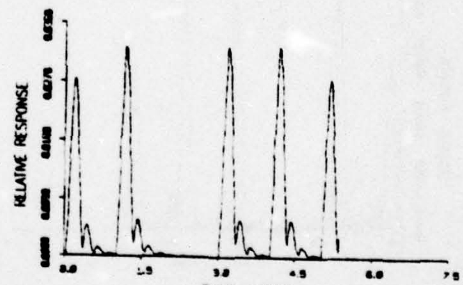
(c) $\Delta f/B_r = 1$



(d) $\Delta f/B_r = 2$

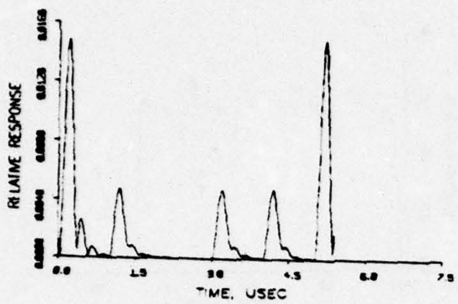


(e) $\Delta f/B_r = 4$

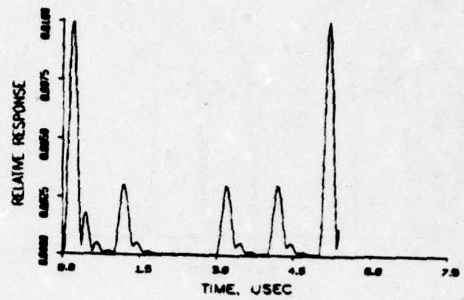


(f) $\Delta f/B_r = 6$

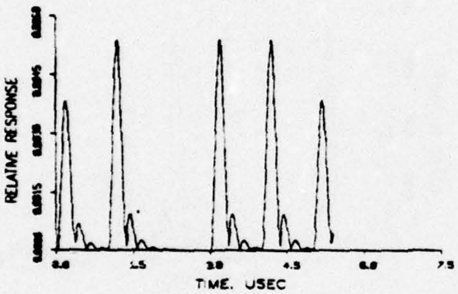
Figure E-4. Responses to DS signal showing the effects of Δf when $B_r\tau = 5$ ($\tau = 1$ μs, signal duration = 5 μs).



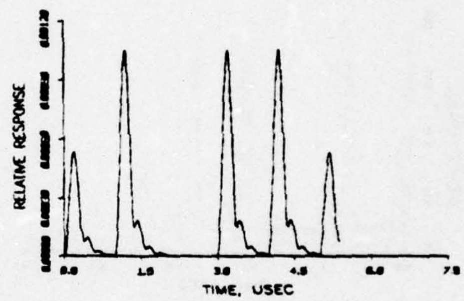
(g) $\Delta f/B_r = 9$



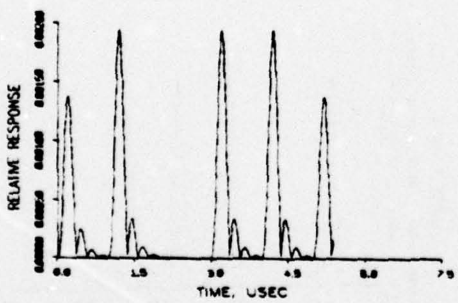
(h) $\Delta f/B_r = 11$



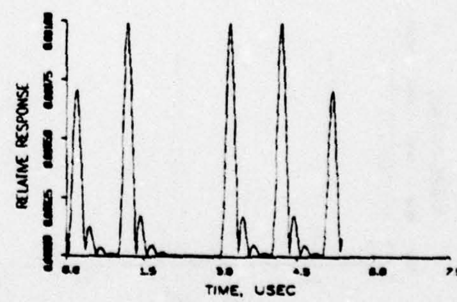
(i) $\Delta f/B_r = 15$



(j) $\Delta f/B_r = 19$

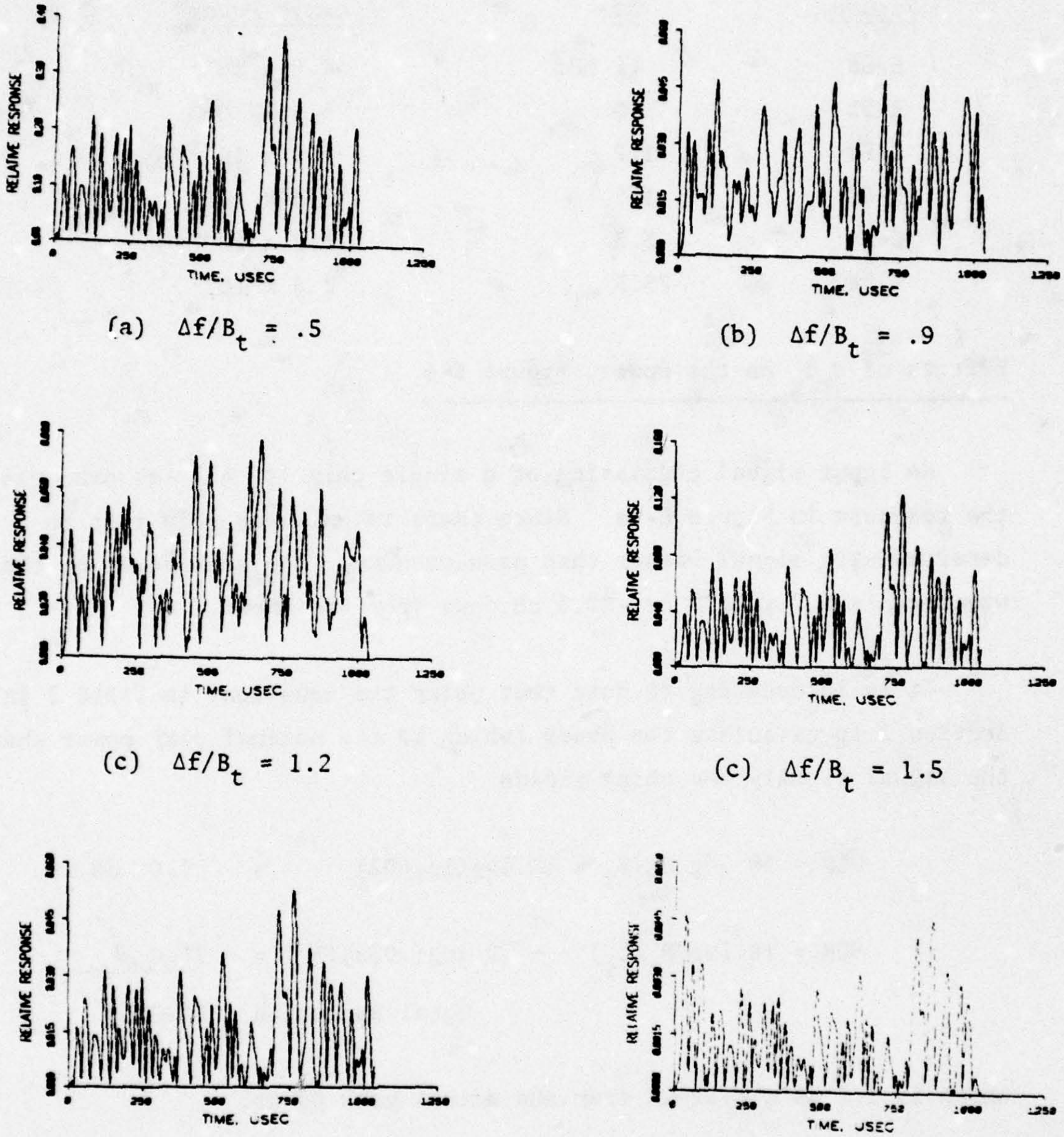


(k) $\Delta f/B_r = 25$



(l) $\Delta f/B_r = 35$

Figure E-4. (continued)



(a) $\Delta f/B_t$ = .5

(b) $\Delta f/B_t$ = .9

(c) $\Delta f/B_t$ = 1.2

(c) $\Delta f/B_t$ = 1.5

(e) $\Delta f/B_t$ = 3.5

(f) $\Delta f/B_t$ = 25.5

Figure E-5. Responses to DS signal showing the effects of Δf when $B_r\tau$ = .05 (τ = 1 μs, signal duration = 1000 μs).

Figure	Δf	Average Power[a]
E-5a	.5 MHz	1.6×10^{-2}
E-5b	.9	5.5×10^{-4}
E-5c	1.2	1.2×10^{-3}
E-5d	1.5	1.8×10^{-3}
E-5e	3.5	3.2×10^{-4}
E-5f	25.5	2.8×10^{-6}

Effects of $B_r\tau_g$ on the Power, Figure E-6

An input signal consisting of a single chip (τ_g = 1 μs) produces the response in Figure E-6a. Since there is only one chip this is a deterministic signal rather than pseudorandom. The peak power of the waveform is 5.2×10^{-6} or -52.8 dB down from the input.

It is interesting to note that using the equations in TABLE 2 in Section 2 to calculate the power (which is the nominal peak power when the signal is only one chip) yields

$$\text{OTR} = 10 \log B_t/B_r = 10 \log(1/.002) = 27.0 \text{ dB}$$

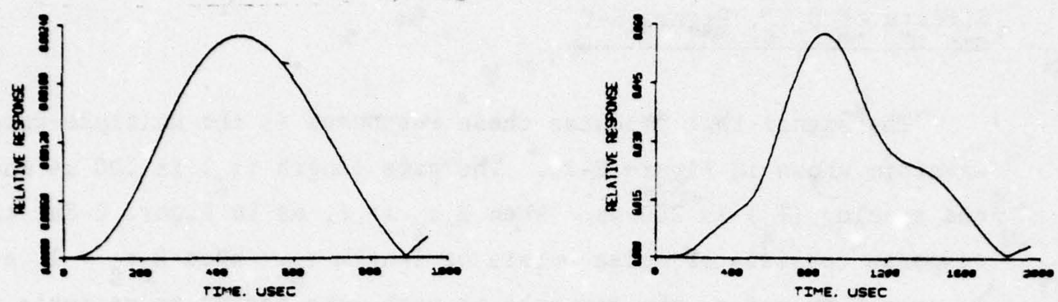
$$\text{SDR} = 10 \log(B_r/\tau_g)^{-1} = 10 \log(.002)^{-1} = \underline{27.0}$$

$$\text{Total Rejection} \quad 54.0 \text{ dB}$$

which is 1.2 dB different from the actual peak power.

The input signal that produces the response in Figure E-6b is a 1000-chip (τ_g = 1000 μs) pseudorandom sequence with a code length (N_c) of 2047. The peak power of this waveform is 5.7×10^{-3} or 24.9 dB

[a]The input power is one unit of power.



(a) $\tau_g = 1\ \mu s$ (b) $\tau_g = 1000\ \mu s$

$B_r\tau_g = .002$ $B_r\tau = 2$

Figure E-6. Response to DS signal consisting of a single gate showing effects of gate length τ_g ($\tau = 1\ \mu s$, $B_r = .002$. $\Delta f = 1$).

down from the input. Using the equations from TABLE 2 yields

$$\begin{aligned} \text{OTR} &= 27.0 \text{ dB} \\ \text{SDR} &= \underline{\ \ 0\ \ } \\ \text{Total Rejection (Expected)} &= 27.0 \text{ dB} \end{aligned}$$

The expected power is about 2 dB greater than that of the waveform in the plot. The amplitudes of the response waveforms vary pseudorandomly, and the equations given in Section 2 yield the expected power, which may be different from the actual power in a given waveform by as much as about 8 dB.

<u>Effects of $B_t\tau_g$ on Waveforms, Figure E-7</u>

The signal that produces these responses is a 100-chip pseudorandom sequence. Note that when $B_r\tau_g < 1$ the response is an impulse with a nominal length of $1/B_r$, as in Figures E-7c and d.

Effects of $B_r T_g$, Figure E-8

The signal that produces these responses is the multiple-gated waveform shown in Figure E-1b. The gate length (τ_g) is 100 μs and the spacing (T_g) is 200 μs. When $B_r \tau_g >> 1$, as in Figure E-8a, the response consists of noise bursts of length τ_g. When $B_r \tau_g \rightarrow 1$, as in Figures E-8b and c, the response to each gate begins to resemble an impulse. When $B_r T_g < 1$, as in Figure E-8d, the impulse responses overlap one another so that the response takes on a noise-like appearance.

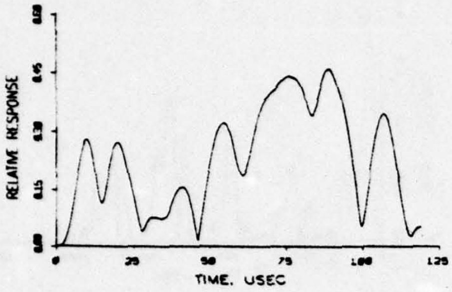
Effects of Frequency Hopping (Bandpass Centered on F_1), Figure E-9

The FH/DS signal that produces these responses is configured as shown in Figure E-1c. The dwell time (τ_f) on each frequency is 100 μs. The five frequencies are spaced 1 MHz apart with the sequence shown in the figure. This spacing yields a spectrum configuration similar to that shown in Figure 15 in Section 2. The bandpass filter is centered on F_1 so that $\Delta f_j = 0, 1, 2, 3$, and 4, where $\Delta f_j = f_j - f_r$.

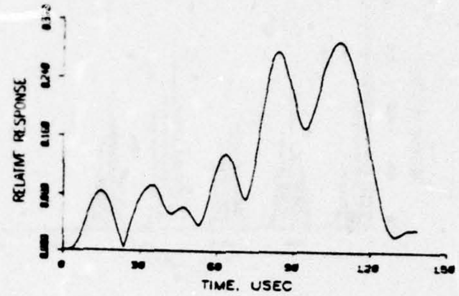
When $B_{r\ f} >> 1$, as in Figures E-9a and b, the response to F_1 consists of noise bursts. The responses to the other four frequencies are especially low because the nulls in the spectra associated with these frequencies are coincident with the center of the passband.

Effects of Frequency Hopping (Bandpass Centered Between F_1 and F_2), Figure E-10.

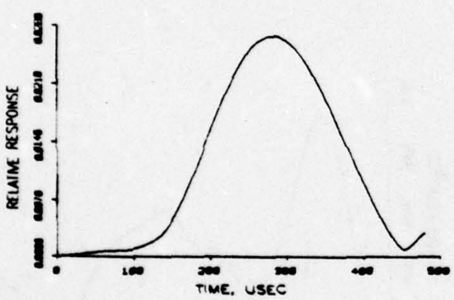
The FH/DS signal for these responses is the same as for Figure E-9. The difference is that the filter is centered midway between F_1 and F_2 so that $\Delta f_j = -.5, .5, 1.5, 2.5$ MHz. With this configuration the responses to F_1 and F_2 have the same expected values.



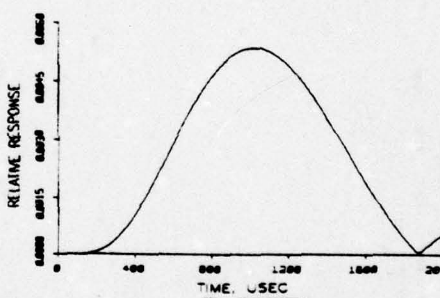
(a) $B_r \tau_g = 10$
$B_r = .1$ MHz



(b) $B_r \tau_g = 5$
$B_r = .05$ MHz

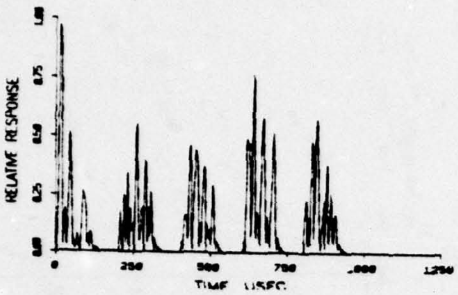


(c) $B_r \tau_g = .5$
$B_r = .005$ MHz



(d) $B_r \tau_g = .1$
$B_r = .001$ MHz

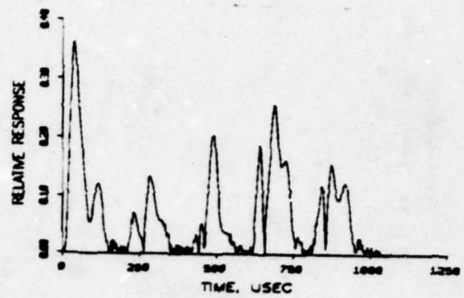
Figure E-7. Response to DS signal consisting of a single gate showing effects of the filter bandwidth, B_r (τ = 1 μs, τ_g = 100 μs, $\Delta f = 0$, signal duration = 100 μs).



(a) B_r = MHz

$B_r\tau_g = 10$

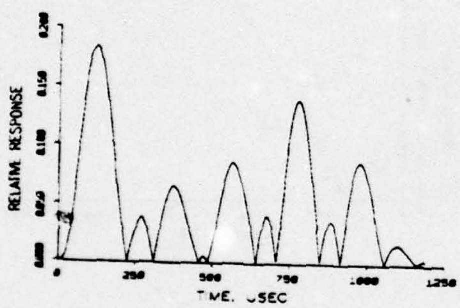
$B_rT_g = 20$



(b) $B_r = .03$ MHz

$B_r\tau_g = 3$

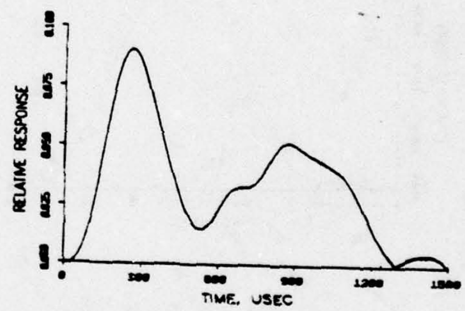
$B_rT_g = 6$



(c) $B_r = .01$ MHz

$B_r\tau_g = 1$

$B_rT_g = 2$

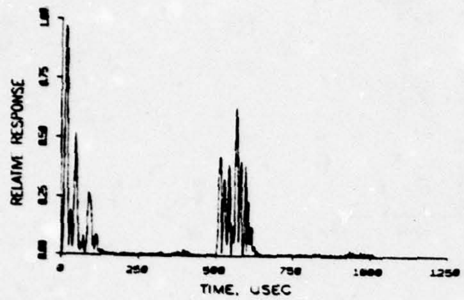


(d) $B_r = .004$ MHz

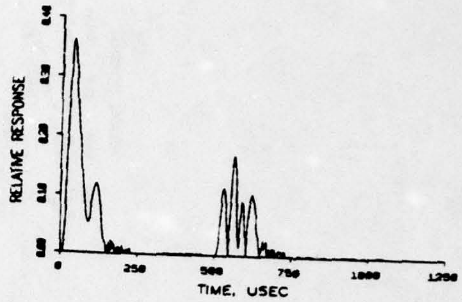
$B_r\tau_g = .4$

$B_rT_g = .8$

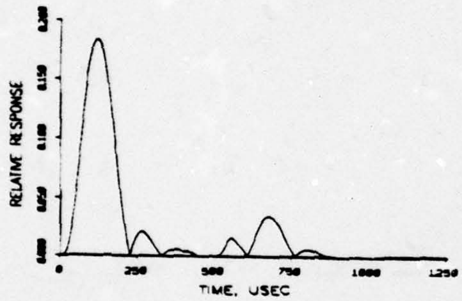
Figure E-8. Response to multiple-gated DS signal showing the effects of the filter bandwidth, B_r ($\tau = 1$ μs, $\tau_g = 100$ μs, $T_g = 200$ μs, $\Delta f = 0$. signal duration = 1000 μs).



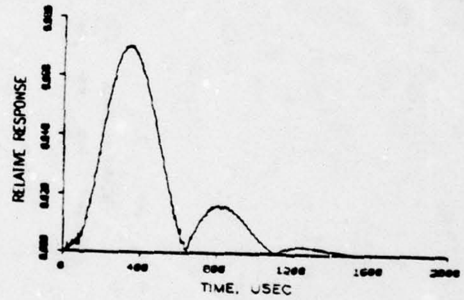
(a) $B_r \tau_f = 10$



(b) $B_r \tau_f = 3$

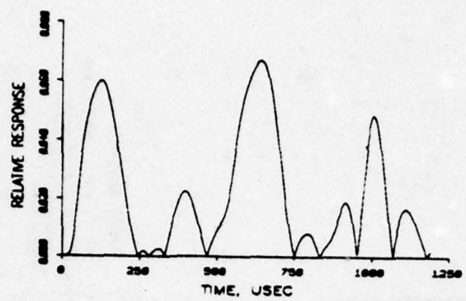


(c) $B_r \tau_f = 1$

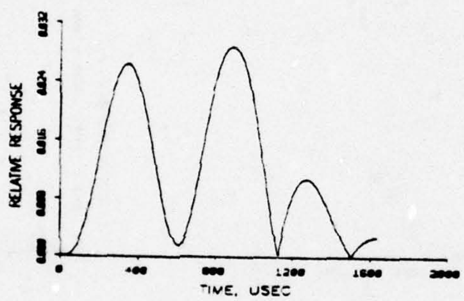


(d) $B_r \tau_g = .3$

Figure E-9. Response to a FH/DS signal ($\tau = 1$ μs, $\tau_f = 100$ μs, $T_f = 200$, $N_f = 5$, $\Delta f_j = 0, 1, 2, 3, 4,$ MHz where j = 1, 2, 3, 4, 5; signal duration = 1000 μs).



(a) $B_r \tau_f = 1$



(b) $B_r \tau_f = .3$

Figure E-10. Response to an FH/DS signal (same as Figure E-8 except Δf_j = -.5, 1.5, 2.5, 3.5 MHz).

APPENDIX F

SPECTRAL CHARACTERISTICS OF SS SIGNALS

BACKROUND

Two kinds of spectra are of interest here: the power-density spectrum (watts/Hz), which is a continuous function, and the power spectrum (watts), which is a discrete function.

Power Density Spectrum

The power-density spectrum of the SS signal consists of a center lobe (or in-band region) and sidelobes (or out-of-band region) as shown in Figure F-1. This kind of spectrum is used in Option No. 3 (see Section 2) for calculating OFR.

In EMC (electromagnetic compatibility) analysis, especially when the out-of-band regions of the spectrum are of primary interest, it is often meaningful, as well as convenient, to describe the spectrum by its bounds, which is a simple monotonic decreasing function that is essentially tangent to the major lobes of the spectrum, as shown in Figure F-1. Option No. 2 entails the use of bounding functions for calculating OFR.

Power Spectrum

A power spectrum, which is produced when a signal is periodic and infinite in length, is customarily represented by lines. When certain conditions exist, the spectrum of an SS signal can be approximated by a line spectrum for the sake of convenience, as will be explained.

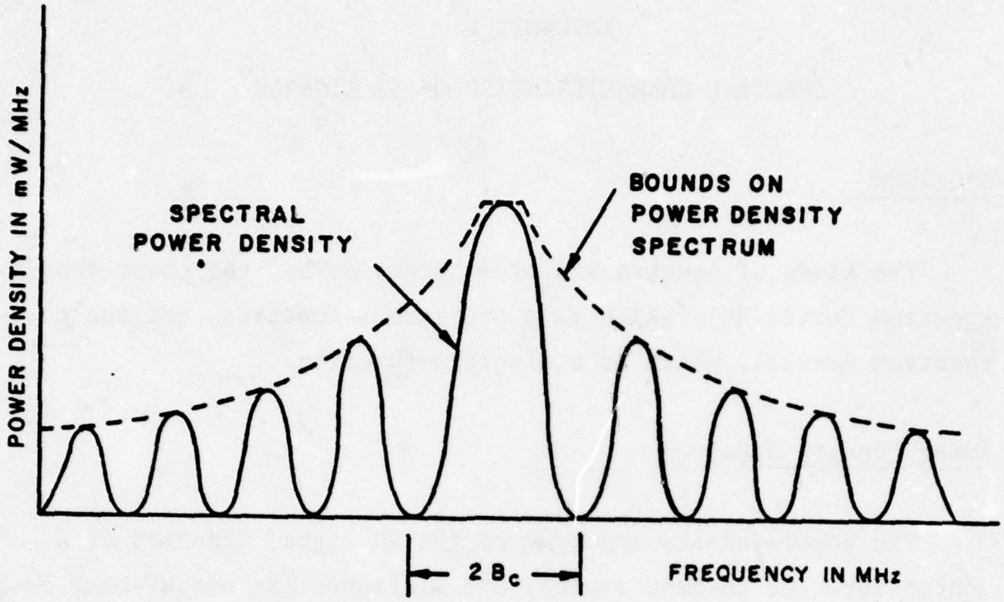


Figure F-1. The power-density spectrum of an SS signal, PSK.

In SS transmitters using the DS technique, the baseband or information signal is put into digital form and modulo-2 added to the code sequence. The resulting digital signal, which Dixon (Reference 1) refers to as (information $\oplus$ code), is used to modulate the RF carrier. The spectrum of the (information $\oplus$ code) is determined by the convolution of the spectra of the code sequence and the information signal.

In theory, a pseudorandom, or maximal-length, code sequence produces a line spectrum (Reference 14). The spacing between lines in B_c/N_c where B_c is the chip rate of the code and N_c is the number of chips in the sequence. For pseudorandom or maximal-length sequences, $N_c = 2^n - 1$, where n is the number of bits in the code-generating shift register.

For example, consider a range-measuring SS system with a transmission that is much longer than (N_c/B_c) μs and an information signal consisting of an oscillator output; i.e., $B_i = 0$ and the spectrum is a single line. Convolving this line with the line spectrum of the code sequence yields the spectrum of the (information $\oplus$ code) which turns out to be a line spectrum that is the same as the spectrum of the code sequence, such as shown in Figure F-2a.

Now if the information signal were changed so that $B_i > 0$ and if it were random, it would have a continuous spectrum. Convolving this spectrum with the line spectrum of the code sequence would yield a continuous spectrum, such as shown in Figures F-2b and c. When $B_c/B_i \gg N_c$ and the transmission time is long compared to (N_c/B_c) μs, the spectrum has distinguishable lobes that are the residue of the lines in spectrum of the code sequence, as shown in Figure F-2b. As B_i is increased, the spectrum takes on the shape shown in Figure F-2c.

The generation of pulsed FM signals does not involve a periodic process, and the spectrum will not have lines unless the baseband signal happens to have a periodic component.

Whether the lines in the spectrum must be considered in an analysis depends on the duration of the SS signal and the bandwidths of the baseband signal and victim receiver. The procedures given in Section 2 were derived using the type of spectrum that is appropriate for each bandwidth condition.

SPECTRAL FUNCTIONS

The spectral function for each type of modulation is given here following a general explanation of the nomenclature used.

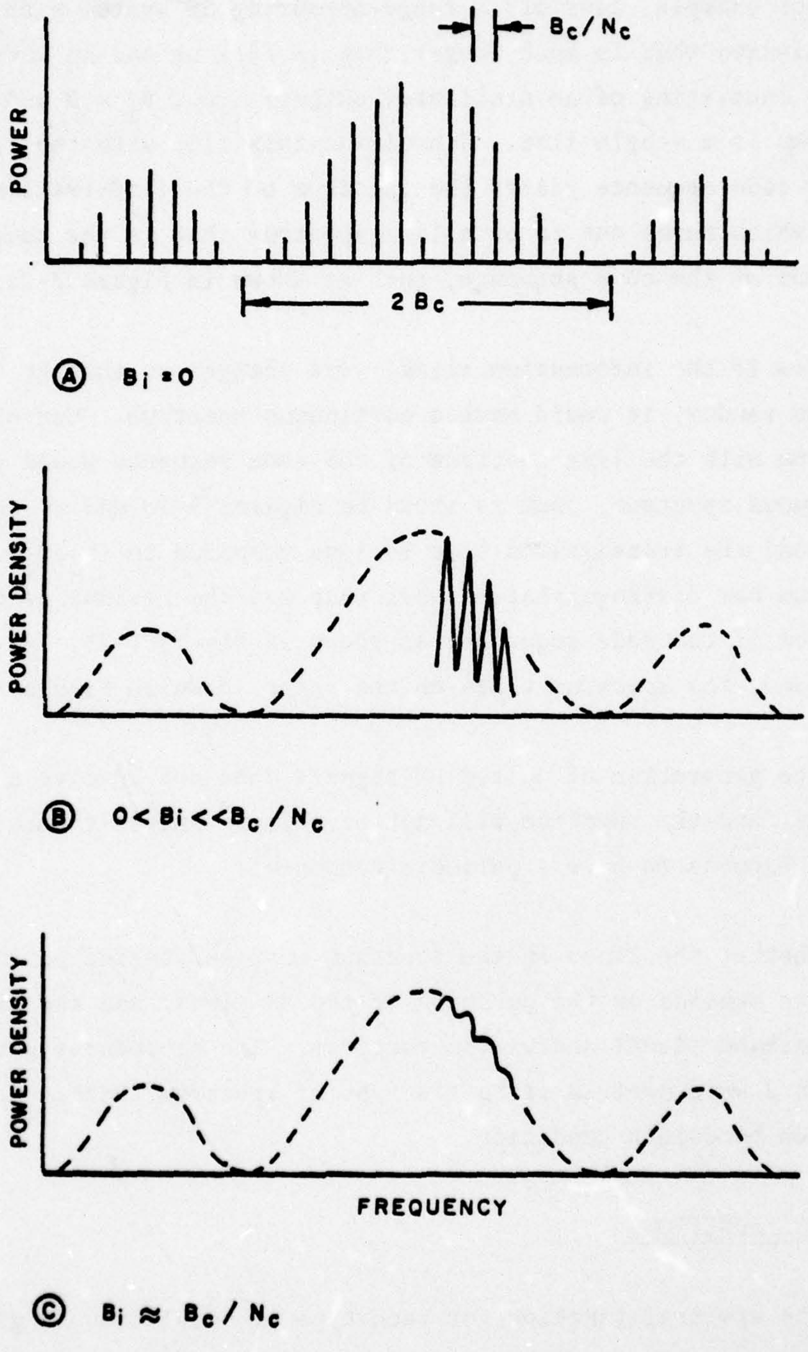


Figure F-2. Spectrum of information ⊕ code, PSK.

The *power density spectrum* (mW/MHz) equation is

$$\phi\{\Delta f\} = (p_t/B_t)\ s_p\ f_t\{\Delta f\} \tag{F-1}$$

where p_t is the signal power (mW), B_t is the bandwidth (MHz), s_p is the peaking factor, and $f_t\{\Delta f\}$ is the normalized spectral density function. For example, the spectrum for an MSK signal is

$$\phi\{\Delta f\} = p_t \tau \frac{16}{\pi 2} \left[\frac{\cos 2\pi\Delta f\tau}{1 - (4\tau\Delta f)^2}\right]^2$$

Thus, for MSK the terms used in Equation F-1 are:

$$(p_t/B_t) = (p_t\tau)$$
$$s_p = 16/\pi^2$$
$$f_t\{\Delta f\} = \left[\frac{\cos(2\pi\Delta f\tau)}{1-(4\tau\Delta f)^2}\right]^2$$

The power density spectrum expressed in dBm/MHz is

$$\phi\{\Delta f\} = 10 \log(p_t/B_t) + S_p + F_t\{\Delta f\} \tag{F-3}$$

$$S_p = 10 \log(s_p) \tag{F-4}$$

$$F_t\{\Delta f\} = 10 \log f_t\{\Delta f\} \tag{F-5}$$

The normalized spectrum is never positive; i.e., $F_t\{0\} = 0$ and $F_t\{\Delta f\} \leq 0$ for $|\Delta f| > 0$.

The *spectrum of a state change* in the signal is used to calculate the OFR when $B_t/B_r < c_1$ and $|\Delta f| > B_r/2$, as explained in APPENDIX B. When this condition exists, each change in signal state causes an impulse response. The peak of the impulse, and hence the OFR, is calculated using the spectrum produced by a single change in state. This spectrum is denoted by $F_\delta\{\Delta f\}$.

For example, for a PSK signal the spectrum of the ramp-step function that describes the assumed change in amplitude as the phase changes by 180 degrees is used in calculating the OFR.

The relative magnitude and probability of the impulse response produced at the end of each chip depends on the kind of modulation. For example, with PSK the relative magnitudes are 0 and 1, each having a probability of .5, whereas for QPSK the relative magnitudes are 0, .707, and 1, with probabilities of .25, .5, and .25 respectively. To account for this behavior a random variable (y) is included in the equation for $F_\delta\{\Delta f\}$, as will be shown later.

The *bounds on the spectral functions*, denoted by $\tilde{F}_t\{\Delta f\}$ and $F_\delta\{\Delta f\}$, are defined as follows:

$$\tilde{F}_t\{\Delta f\} \geq \left|F_t\{\Delta f\}\right| \quad \text{(F-6)}$$

$$\tilde{F}_\delta\{\Delta f\} \geq \left|F_\delta\{\Delta f\}\right| \quad \text{(F-7)}$$

The spectral functions for each type of modulation are given in the paragraphs below.

<u>PSK and QPSK Spectral Functions</u>

$$S_p = 0 \text{ dB} \quad \text{(F-8)}$$

$$F_t\{\Delta f\} = 10 \log \left[\frac{\sin(\pi\Delta f\tau)\ \sin(2\pi\Delta f\delta)}{2\tau\delta(\pi\Delta f)^2}\right]^2 \quad \text{(F-9)}$$

$$F_\delta\{\Delta f\} = \begin{cases} \text{not used when } |\Delta f| < B_r/2 \\ 10 \log \left[\dfrac{y \sin(2\pi\Delta f\delta)}{2\delta(\pi\Delta f)^2}\right]^2, & |\Delta f| \geq B_r/2 \end{cases} \quad \text{(F-10)}$$ [27]

For PSK, $\begin{cases} y = 0 \text{ or } 1 \\ p(0) = .5,\ p(1) = .5 \end{cases}$

For QPSK, $\begin{cases} y = 0,\ .707, \text{ or } 1 \\ p(0) = .25,\ p(.707) = .5,\ p(1) = .25 \end{cases}$

where $p(y)$ is the probability distribution of the random variable y. The bounding function $\tilde{F}_t\{\Delta f\}$ for PSK and QPSK is shown in Figure 10 in Section 2.

OQPSK Spectral Functions[28]

$$S_p = 3 \text{ dB} \quad \text{(F-11)}$$

$$F_t\{\Delta f\} = 10 \log \left[\frac{\sin(2\pi\Delta f\tau)\ \sin(2\pi\Delta f\delta)}{4\tau\delta(\pi\Delta f)^2}\right]^2 \quad \text{(F-12)}$$

$$F_\delta\{\Delta f\} = \begin{cases} \text{not used when } |\Delta f| < B_r/2 \\ 10 \log \left[\dfrac{y \sin(2\pi\Delta f\delta)}{2\delta(\pi\Delta f)^2}\right]^2, & |\Delta f| \geq B_r/2 \end{cases} \quad \text{(F-13)}$$

$y = 0 \text{ or } .707$

$p(0) = .5,\ p(.707) = .5$

The bounding function $\tilde{F}_t\{\Delta f\}$ for OQPSK is shown in Figure 10 in Section 2.

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[27] Papoulis, A., *The Fourier Integral and Its Applications*, McGraw-Hill 1962, p. 40-41.

[28] Gronemeyer, S. A., and McBride, A. L., *MSK and Offset QPSK Modulation*, IEEE Transaction on Communications, Vol. COM-24, No. 8, Aug, 4, 1976, p. 809-820.

MSK Spectral Functions (Reference 27)

$$S_p = 2 \text{ dB} \tag{F-14}$$

$$F_t\{\Delta f\} = 10 \log \left[\frac{\cos 2\pi\Delta f\tau}{1-(4\tau\Delta f)^2}\right]^2 \tag{F-15}$$

$$F_\delta\{\Delta f\} = \begin{cases} F_a, & |\Delta f| \leq (1/4\tau)\sqrt{1 + (4\tau B_r/\pi)} \\ 10 \log \left[\dfrac{y4_\tau}{\pi\{1-(4\tau\Delta f)^2\}}\right]^2, & \text{otherwise} \end{cases} \tag{F-16}$$

$$y = 0 \text{ or } 1$$

$$F_a = 20 \log(1/B_r)$$

$$p(0) = .5,\ p(1) = .5$$

The bounding function $\tilde{F}_t\{\Delta f\}$ for MSK is shown in Figure 11 in Section 2.

PFM Spectral Function

$$S_p = 0 \text{ dB} \tag{F-17}$$

The functions $F_t\{\Delta f\}$ and $F_\delta\{\Delta f\}$ involve Fresnel integrals,[29] which are too complicated for easy hand computation, and are not given here. The bounding function $\tilde{F}_t\{\Delta f\}$ for PFM is given in Figure 12 in Section 2.

* * *

[29] Newhouse, P. D., "Bounds on the Spectrum, of a CHIRP Pulse," *IEEE Transactions on Electromagnetic Compatibility*, Vol. EMC-20, No. 1, February 1973, p. 27-33.

GRAPH OF CENTER LOBE

The shape and dimensions of the center lobe, which includes the in-band region of the spectrum, are of special interest because the center lobe includes the peak of the spectrum and is the region that is most likely to cause interference.

The center lobes of the power-density spectra produced by DS signals involving binary phase-shift keying (PSK), quatrature phase-shift keying (QPSK), and minimum-frequency-shift keying (MSK), sometimes called continuous-phase-shift keying, are shown in Figure F-3. The center lobe of the spectrum produced by the pulsed-FM (PFM) technique is shaped by filtering so that it is essentially rectangular, which is the ideal shape, as shown in Figure F-4.

Several of the dimensions of the center lobes of the various spectra are listed in TABLE F-1.

SPECTRAL FUNCTIONS FOR FREQUENCY-HOPPING TRANSMITTERS

Frequency-hopped signals involving N_f frequencies with a hopping rate of B_h(Mh/s) are treated as N_f individual signals. Each of these signals is gated, with a gate length τ_f ($\tau_f = 1/B_h$), an average spacing T_f, and a gate rise/fall time δ_f. The frequency sequence is assumed to be pseudorandom so that the spacing of the gates associated with any one frequency is Poisson distributed. Thus, the average spacing between the gates associated with any one frequency is

$$T_f = N_f \tau_f$$

The gate rise/fall time is assumed to be

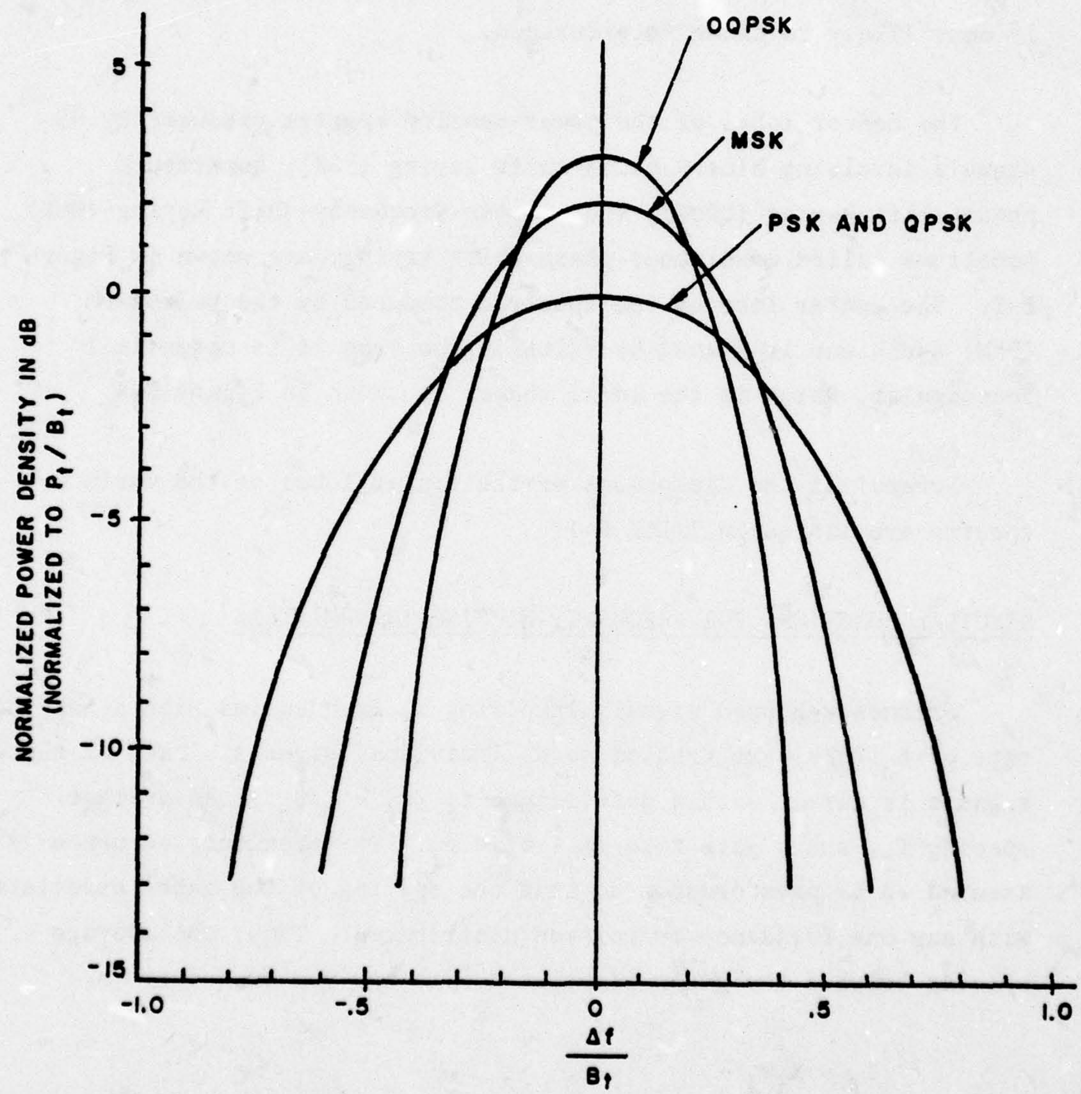

Figure F-3. Centerlobes of power-density spectra of several types of SS signals, normalized to p_t/B_t.

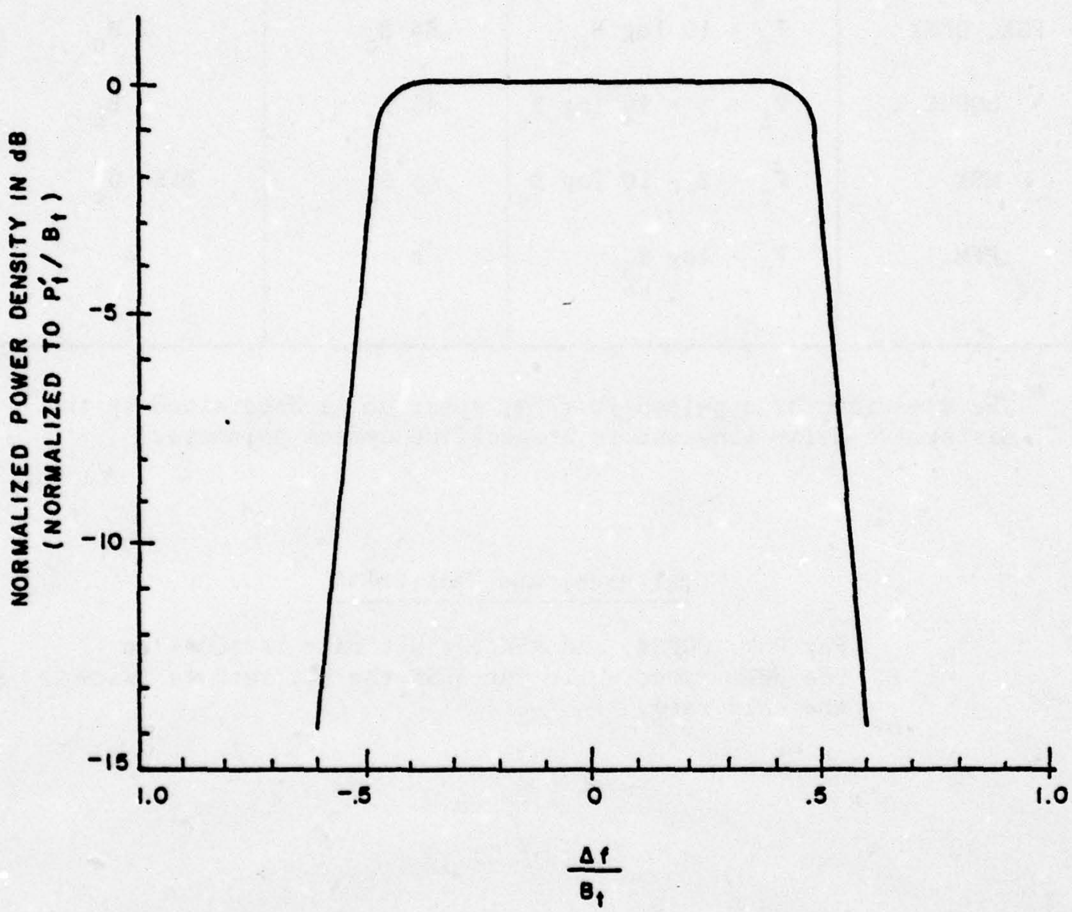

Figure F-4. Envelope of the centerlobe of power-density spectrum of pulsed-FM (PFM) SS signal, normalized to p_t/B_t.

TABLE F-1

FORMULAS FOR
PEAK SPECTRAL-POWER DENSITIES AND BANDWIDTHS

TYPE OF MODULATION	PEAK SPECTRAL POWER DENSITY (dBm/MHz)	3 dB BANDWIDTH (B_3) (MHz)	BANDWIDTH BETWEEN NULLS FOR CENTER LOBE (MHz)
PSK, QPSK	$P_t - 10 \log B_c$	$.88\ B_c$	$2\ B_c$
OQPSK	$P_t + 3 - 10 \log B_c$	$.45\ B_c$	B_c
MSK	$P_t + 2 - 10 \log B_c$	$.60\ B_c$	$1.5\ B_c$
PFM	$P_t - \log B_3$	a	a

[a] The bandwidth of a pulsed-FM (PFM) spectrum is determined by the dispersive delay line and is a specified system parameter.

Bit Rates and Chip Rates

For PSK, OQPSK, and MSK the bit rate is equal to the chip rate, while for QPSK the bit rate is twice the chip rate.

$$\delta_f = \tau_f/10$$

unless a better estimate is available.

The information, or baseband, signal is assumed to be in digital form with a bandwidth B_i (MHz) and a rise/fall time δ_i. Unless a better estimate is available, δ_i is related to B_i as follows:

$$\delta_i = \tau_i/10$$

where

$$\tau_i = 1/B_i$$

For frequency-hopping signals, the analyst may choose to calculate the INR for the one frequency that happens to be closest to the tuned frequency of the victim receiver, or for each of the frequencies if a more thorough analysis is to be performed. The application of the INR Equation to frequency-hopped signals is explained in Section 2.

The OFR is calculated for one or each of the individual frequencies, depending which of the above approaches is used. The spectra of the signals are identical. However, the jth spectrum is centered at F_j as illustrated in Figures 14 and 15 in Section 2.

For a frequency-hopped (FH) signal, the characteristics of a spectrum are determined primarily by the information signal when $B_i/B_h >> 1$, or by the frequency-hopping rate when $B_i/B_h << 1$. For a frequency-hopped hybrid (FH/DS) signal the B_c/B_h is assumed to be always much greater than the unity; thus, the spectrum of such a signal is determined primarily by the DS parameters. The spectral functions for both FH and FH/DS signals are given on the following pages.

For FH

When $B_i/B_h \geq 1$ (or $B_i\tau_f \geq 1$), the information signal spectral functions are used.

$F_i\{\Delta f_j\}$: use Equation F-9[a] (F-18)

$F_{\delta i}\{\Delta f_j\}$: use Equation F-10[a] (F-19)

$\tilde{F}_i\{\Delta f_j\}$: Figure 10[a] (F-20)

When $B_i/B_h < 1$ (or $B_i\tau_f < 1$), the spectral functions used are determined by the frequency-hopping parameters

$F_f\{\Delta f_j\}$: use Equation F-9[b] (F-21)

$F_{\delta f}\{\Delta f_j\}$: use Equation F-10[b] (F-22)

$\tilde{F}_f\{\Delta f_j\}$: use Figure 10[b] (F-23)

Notes:

[a]Except that Δf_j is substituted for Δf, τ_i for τ, and δ_i for δ.

[b]Except that Δf_j is substituted for Δf, $2\tau_f$ for τ, $\delta_f/2$ for δ, and y is unity.

For FH/DS

$F_h\{\Delta f_j\}$: use Equation F-9, F-12, or F-15[a,b] (F-24)

$F_{\delta h}\{\Delta f_j\}$: use Equation F-10, F-13, or F-16[a,b] (F-25)

$\tilde{F}_h\{\Delta f_j\}$: Figure 10, 11, or 12[a,c] (F-26)

Notes:

[a]Except that Δf_j is substituted for Δf.

[b]Depending on whether the DS signal is PSK, QPSK, OQPSK, or MSK; equations are not given for PFM.

[c]Depending on whether the DS signal is PSK, QPSK, OQPSK, MSK, or PFM.

APPENDIX G

SAMPLE CALCULATIONS

Some examples are worked out here to illustrate the application of the analytical procedures given in Section 2. The calculations follow the "Guidelines for Using Procedures Given In This Section," page 14.

GIVEN PARAMETER VALUES

Unless indicated otherwise, each of the examples uses these parameter values.

Interfering Transmitter

Type of modulation: binary PSK chip rate = B_c = 10 megachips/s (Mc/s)

Carrier frequency = 1800 MHz

Power = P_t = 40 dBm (10 W)

Antenna gain = G_t = 3 dBi

Transmitter filter bandwidth = B_F = 10 MHz
(number of pole pairs = p = 4)

Transmitter noise = N/C = -140 dBm/Hz

The signal is not gated (assume $\tau_g = \infty$)

Code length = $N_c = 2 \times 10^9$

Rise time: since the rise time is not given, assume that $\delta = .1/B_c$

Victim Receiver Parameters

IF 3 dB bandwidth = B_r = .025 MHz
(number of pole-pairs = p = 6)

Antenna gain = G_r = 5 dB

Noise Figure = F_r = 10 dB

Allowable reduction in S/N as a result of the interference is C = 1 dB

Propagation Path Loss

Use the curve in Figure G-2 that is a plot for a smooth earth with antenna heights of 10 feet for one antenna and 100 feet for the other.

SAMPLE CALCULATIONS

Problem No. (1) Determine The Type of Response Waveform

The type of waveform depends on the bandwidth ratio, B_t/B_r. For PSK, we have the relationship $B_t = B_c$ (Equation 6, Section 2 of this report). Thus,

$$B_t/B_r = 10/.025 = 400$$

which satisfies the condition

$$1 < B_t/B_r < N_c$$

Therefore, we can use the formulas in TABLE 2 (Section 2 of this report). The transmitted SS signal is not gated in this example; thus, as indicated in the table, the response is noise-like.

Answer: The response waveform is noise-like.

Problem No. (2) Determine The Required Distance Separation When $|\Delta f| = 0$.

This entails calculating the path loss (L_p) that is required to yield a value of INR that is equal to INR_t. Using Equation 1 and 2 (Section 2) and substituting INR_t for INR gives the following relationship, which will be used to calculate the required path loss.

$$L_p = P_t + G_t + G_r - N_r - OTR - OFR - SDR - INR_t \quad \text{(G-1)}$$

The values for P_t, G_t, and G_r are given, but the remaining terms must be calculated as follows:

$$N_r = -114 + F_r + 10 \log B_r \quad \text{(See Equation 2)}$$
$$= -114 + 10 + 10 \log .025 = -120 \text{ dBm}$$

$$OTR = -S_p + 10 \log B_t/B_r \quad \text{(TABLE 2)}$$

For PSK, $S_p = 0$ dB, therefore (See Equation 5)

$$OTR = -0 + 10 \log 10/.025 = 26 \text{ dB}$$

For this example, $\Delta f = 0$ and $\tau_g = \infty$ (i.e., the signal is ungated). Therefore, OFR = 0 dB and SDR = 0 dB, as indicated in TABLE 2.

An appropriate value must be selected for INR_t. As explained earlier (Section 2), INR_t depends on the allowed change (C) in the S/N. The value of C for this example is given as 1 dB. Thus,

$$INR_t = 10 \log(-1 + 10^{C/10}) \quad \text{(See Equation 15)}$$
$$= 10 \log(-1 + 10^{1/10})$$
$$= -6 \text{ dB}$$

All of the parameter values needed to calculate L_p are either given or have now been determined. In summary,

P_t = 40 dBm	OTR = 26 dB
G_t = 3 dBi	OFR = 0 dB
G_r = 3 dBi	SDR = 0 dB
N_r = -120 dBm	INR_t = -6 dB

Substituting these values in Equation G-1 yields L_p = 148 dB.

Referring to Figure G-1, we see that for a required path loss of 148 dB the distance separation must be 17 miles.

Answer: when $\Delta f = 0$, the required distance separation is 17 miles.

Problem No. (3). Determine the Required Frequency Separation When The Distance Separation Is 2 Miles (Using Calculated Spectral Bounds $\tilde{F}_t\{\Delta f\}$

Referring to the plot of path loss vs distance in Figure G-1, we see that a distance of 2 miles results in a path loss of 102 dB. Equation G-1 can be rearranged as follows to calculate the required OFR.

$$\begin{aligned} OFR &= P_t + G_t + G_r - L_p - N_r - OTR - SDR - INT_t \qquad \text{(G-2)} \\ &= 40 + 3 + 5 - 102 - (-120) - 26 - 0 - (-6) \\ &= 46 \text{ dB} \end{aligned}$$

Next we must determine what frequency separation is required for OFR = 46 dB. First we must select an appropriate OFR formula. The SS signal involved here is a direct sequence; thus, we refer to the directory in TABLE 3 which indicates where an appropriate formula can be found.

The bandwidth conditions for this example are as follows:

Signal continuity: uninterrupted ($\tau_g = \infty$)

$$c_1 < B_t/B_r < N_c$$

$$|\Delta f| \geq 0$$

The parameter c_1 is given by Equation 7, Section 2. For PSK, S_p = 0 dB so that c_1 = antilog (S_p/10) = antilog (0) = 1.

The directory indicated that for these conditions the formulas in TABLE 4-c are applicable. From TABLE 4-c, the OFR formula is

$$\text{OFR} = -\tilde{E}_t\{\Delta f\}$$

where

$$\tilde{E}_t = \tilde{F}_t\{\Delta f\} - R_t\{\Delta f\}$$

We must calculate each of the two functions on the right in the above expression. The transmitter-filter attenuation $R_t\{\Delta f\}$ is obtained with Equation 4, and the spectrum bounds $\tilde{F}_t\{\Delta f\}$ using the procedure given in Figure 7.

Thus,

$$R_t\{\Delta f\} = \begin{cases} 0, \text{ for } |\Delta f| \leq 10/2 \\ 20 \times 4 \log (2\Delta f/10), \text{ otherwise} \end{cases}$$

$R_t\{\Delta f\} = 80 \log(.2\Delta f)$ for $|\Delta f| > 5$

Using the procedures in Figure 10, we get the following values that describe the spectral bounds $\tilde{F}_t\{\Delta f\}$:

$$\Delta f_a = 10/\pi = 3.18 \text{ MHz}$$

$$\Delta f_b = 1/(2\pi \times .01), \text{ assuming } \delta = .1/B_c$$
$$15.92 \text{ MHz}$$

$$\Delta f_c = \text{antilog } [(1/2 \log 3.18 \times 15.92) - (-70/40)]$$
$$= \text{antilog } 2.60 = 400 \text{ MHz}$$

where the transmitter noise floor F_n is obtained by

$$F_n = N/C + 60 - S_p + 10 \log B_t$$
$$= -140 + 60 - 0 + 10 \log 10 = -70$$

Using logarithmic graph paper, we plot $\tilde{F}_t\{\Delta f\}$ in Figure G-2 and $R_t\{\Delta f\} - \tilde{F}_t\{\Delta f\}$ in Figure G-3. Thus, we can obtain OFR by

$$\text{OFR} = R_t\{\Delta f\} - \tilde{F}_t\{\Delta f\} = -\tilde{E}_t\{\Delta f\}$$

From Figure G-3, we see that to attain a value of OFR = 46 dB, the frequency separation must be $|\Delta f| = 13$ MHz.

Answer: When the distance separation is 2 miles, the frequency separation must be 13 MHz if we used the spectral bounds $\tilde{F}_t\{\Delta f\}$ to represent the signal spectrum.

Problem No. (4). Determine The Required Frequency Separation When The The Distance Separation Is 2 Miles (Using Calculated Spectral Power Density Function $F_t\{\Delta f\}$

The intent of this problem is to demonstrate the effect of using a more accurate representation of the emission spectrum, i.e. by using $F_t\{\Delta f\}$ in place of the bounds $\tilde{F}_t\{\Delta f\}$ in calculating the OFR. The first step is to plot $F_t\{\Delta f\}$ using Equation F-9 (page 144).

$$F_t\{\Delta f\} = 10 \log \left[\frac{\sin(\pi\Delta f\tau)\ \sin(2\pi\Delta f\delta)}{2\tau\delta(\pi\Delta f)^2}\right]^2$$

For this example, $\tau = 1/B_c = .1\ \mu s$ and $\delta = .1/B_c = .01\ \mu s$. Substituting these values into the above equation and letting $|\Delta f| = 0$ to 20 MHz, we obtain the plot shown in Figure G-4.

In the vicinity of the spectral null at $|\Delta f| = B_c = 10$ MHz, the function $Z_t\{\Delta f\}$ is used as explained in APPENDIX B (page 97). Using Equation B-17b, we get the following equation for $Z_t\{\Delta f\}$:

$$Z_t\{\Delta f\} = 10 \log \left[\frac{\sin(\pi B_{r\varepsilon}/2)\ \sin(\pi\delta 2\Delta f)}{2\tau\delta(\pi\Delta f)^2}\right]^2$$

for $|\Delta f| \approx B_c$. For this example $B_{r\varepsilon} = B_r + 2\varepsilon$, $\delta = .1/B_c$, $\tau = 1/B_c$; $B_r = .025$ MHz, $\varepsilon = .02$ MHz, and $B_c = 10$ megachips/s. A plot of $Z_t\{\Delta f\}$ and $F_t\{\Delta f\}$ for $|\Delta f| = 9$ to 11 MHz is shown in Figure G-5. For purposes of comparison, the spectrum bounds, $\tilde{F}_t\{\Delta f\}$, which were calculated in Problem No. 3 and are shown in Figure G-4, are also plotted in Figure G-5. Note that at $|\Delta f| = 10$ MHz, $\tilde{F}_t\{\Delta f\}$ exceeds $Z_t\{\Delta f\}$ by 40 dB. Thus, in the vicinity of the null in the spectrum, using the more accurate method for calculating OFR (Equation B-15) instead of using OFR equation in TABLE 4(c) (Section 2) yields a calculated value of OFR that is 40 dB larger. Later we will see the effect in terms of distance separation.

The next step is to plot $E_t\{\Delta f\}$, as shown in Figure G-6, where $E_t\{\Delta f\}$ is given by Equation B-16.

$$E_t\{\Delta f\} = -R_t\{\Delta f\} + \max\left[F_t\{\Delta f\},\ Z_t\{\Delta f\},\ F_n\right]$$

The noise floor is at -70 dB, which is well below $Z_t\{\Delta f\}$ at $|\Delta f| = 10$ MHz and can thus be ignored.

For this example,

$$R_r\{\Delta f\} >> -E_t\{\Delta f\}$$

so that Equation B-15 can be replaced by

$$OFR = -E_t\{\Delta f\}$$

Plots of the OFR calculated using the above expression (Curve 1) and of the OFR calculated using the bounds $\widetilde{E}_t\{\Delta f\}$ (from Problem No. 3, Curve 2) are shown in Figure G-7 for comparison.

The required value of OFR is 46 dB, which is the same as in Problem No. 3 (see Equation G-2). From Curve 1 in Figure G-7 we see that OFR = 46 dB when $|\Delta f| = 9.5$ MHz. Note that when the spectral bounds are used to calculate OFR (see Problem 3) the calculated value of $|\Delta f|$ is 13 MHz.

<u>Answer:</u> When the distance separation is 2 miles, the frequency separation must be 9.5 MHz if we use the spectral power density function $F_t\{\Delta f\}$ to represent the signal spectrum.

Problem No. (5). Determine the required distance separation when when $|\Delta f| = 10$ MHz

This will be done two different ways: (1) using the bounds $\tilde{F}_t\{\Delta f\}$ which were calculated in Problem No. 3, and (2) using the spectral power density function $F_t\{\Delta f\}$, which was calculated in Problem No. 4.

Method 1. Using the plot of OFR in Figure G-3, we see that when $|\Delta f| = 10$ MHz, OFR = 34 dB. Next we use Equation G-1 to obtain L_p.

$$L_p = 40 + 3 + 5 - (-120) - 26 - 34 - 0 - (-6) = 114 \text{ dB}$$

Refering to Figure G-1, we find that the required distance is about 4.1 miles.

Method 2. Using the plot of OFR in Figure G-6, we see that at $|\Delta f| = 10$, $-E_t\{\Delta f\} = 74$ dB, which is the OFR at the null. With that value of OFR

$$L_p = 40 + 3 + 5 - (-120) - 26 - 74 - 0 - (-6) = 74 \text{ dB}$$

This value of L_p is too small for the plot in Figure G-1. For short distances, the distance is calculated using the free-space equation with provisions for multipath.

$$D = \text{antilog}\left[\frac{L_p + 6 - 20 \log f - 37}{20}\right]$$

where f is the frequency in MHz and the 6 dB term accounts for multipath. The distance is calculated to be

$$D = \text{antilog}\left[\frac{74 + 6 - 20 \log 1800 - 37}{20}\right] = .08 \text{ miles}$$

Answer: Using Method 1, in which the bounds are used, the required distance is about 4 miles; using Method 2, in which the spectral power density function is used to obtain better accuracy, the required distance is .08 miles.

Problem No. (6). Assuming the Victim Receiver Bandwidth $B_r = 14$, (p = 10), Determine the INR When the Distance is 10 miles and $|\Delta f| = 15$ MHz

First we calculate the bandwidth ratio

$$B_t/B_r = B_c/B_r = 10/14 = .7$$

The value of c_1 is unity (see Problem No. 3). Thus, we have the following condition

$$B_t/B_r < c_1$$
$$|\Delta f| > B_r/2$$

The directory in TABLE 3 indicated that with these conditions the formulas in TABLE 4-b should be used to calculate OTR, OFR, and SDR.

Referring to TABLE 4-b we see that OTR = 0, SDR = 0, and

$$\text{OFR} = \max[0, R_I\{\Delta f\}] \tag{G-3}$$

where

$$R_I\{\Delta f\} = -20 \log \left\{ \text{antilog} \left[\frac{C_{wb} + \tilde{E}_t\{\Delta f\}}{20} \right] + \text{antilog} \left[\frac{-R_r\{\Delta f\}}{20} \right] \right\} \tag{G-4}$$

$C_{wb} = 20 \log B_r/B_c$, for PSK (TABLE 5)

$= 20 \log 14/10 = 2.9$ dB

a good approximation is obtained by

$$R_I\{\Delta f\} \approx \text{Min}\left[(-C_{wb} - \tilde{E}_t\{\Delta f\}),\ R_r\{\Delta f\}\right] \quad \text{(G-5)}$$

The next step is to calculate and plot $R_I\{\Delta f\}$ both ways; i.e., using Equations G-4 and G-5. The first step involves using a plot of $\tilde{E}_t\{\Delta f\}$, which is the bounds on the filtered signal spectrum or the emission spectrum. The bounds $\tilde{E}_t\{\Delta f\}$ for the transmitter were calculated in Problem 3 and $-\tilde{E}_t\{\Delta f\}$ is plotted in Figure G-3. Using this plot to get values of $\tilde{E}_t\{\Delta f\}$ and using Equation 3, in Section 2 to calculate $R_r\{\Delta f\}$, that is

$$R_r\{\Delta f\} = 20 \times 10 \log (2f\Delta/14),$$

the values listed in the table below are obtained.

TABLE G-1

CALCULATED VALUE FOR PROBLEM 6

Δf	$E_t\{\Delta f\}$	C_{wb}	$-R_r\{\Delta f\}$	$R_I\{\Delta f\}$	OFR
7 MHz	-18.5 dB	+2.9 dB	0 dB	-1.3 dB[a]	0 dB[b]
8	-24.0	2.9	-11.6	9.1	9.1
9	-29.5	2.9	-21.8	17.9	17.9
10	-34.0	2.9	-31.0	25.0	25.0
11	-38.0	2.9	-39.3	30.9	30.9
12	-41.0	2.9	-46.8	35.5	35.5
13	-45.0	2.9	-53.8	40.1	40.1
14	-48.0	2.9	-60.2	43.7	43.7
15	-51.0	2.9	-66.2	47.1	47.1

[a]These values were calculated using Equation G-4.

[b]Obtained with Equation G-3.

The results are plotted in Figure G-8. The points are values of OFR obtained using Equations G-3 and G-4.

Figure G-9 shows a comparison of OFR obtained using Equations G-3 and G-4 (the points in Figure G-9) and using Equations G-3 and G-5 (the solid curve). Using the plot in Figure G-9 it can be seen that for $|\Delta f| = 15$ MHz, OFR = 50 dB.

The value of N_r for the receiver is

$$N_r = -114 + F_r + 10 \log B_r$$
$$= -114 + 10 + 10 \log 14 = -92.5 \text{ dBm}$$

The path loss L_p for a distance of 10 miles is 133 dB (obtained from Figure G-1).

The INR can now be calculated,

$$\text{INR} = P_t + G_t + G_r - L_p - N_r - \text{OTR} - \text{OFR} - \text{SDR}$$
$$= 40 + 3 + 5 - 133 - (-93) - 0 - 50 - 0$$
$$= -42 \text{ dB}$$

This is the INR of the waveform caused by the interference. The nature of the response waveform is shown in TABLE 4-b.

Answer: INR = -42 dB.

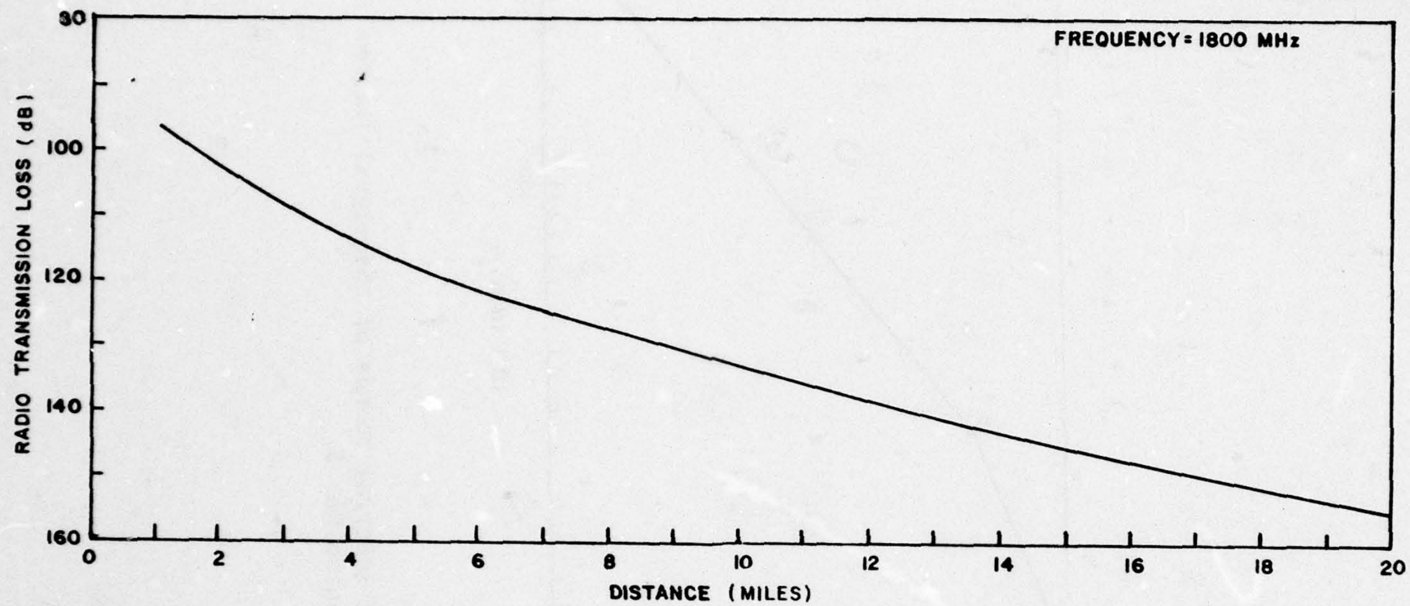

Figure G-1. Basic transmission loss (L_p), for antenna height of 10 feet (transmitter) and 100 feet (receiver).

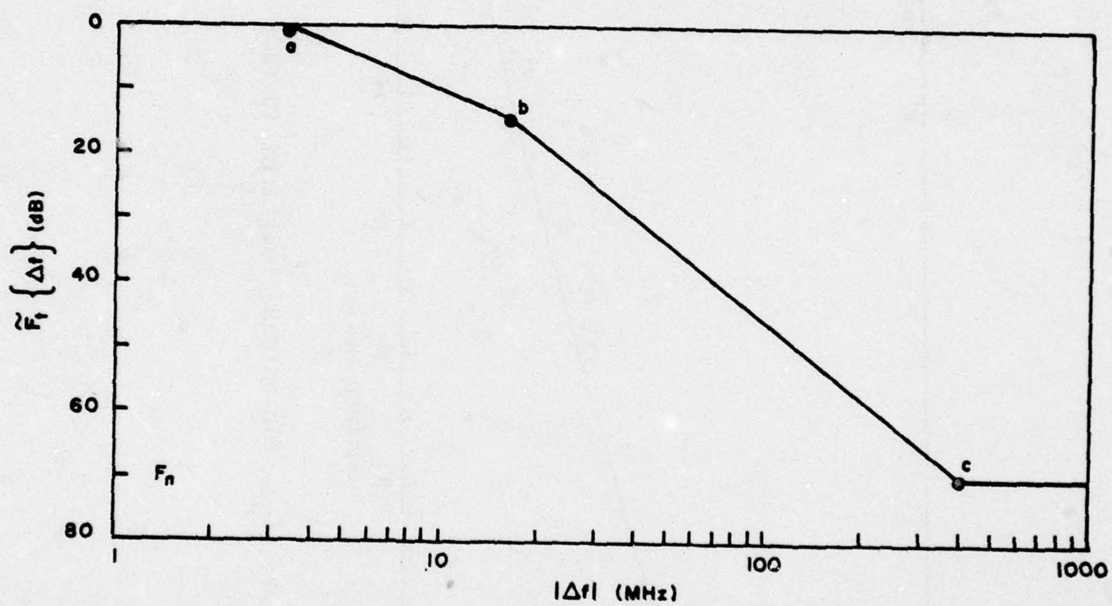

Figure G-2. Plot of spectral bounds of the signal (before filtering) for Problem No. 3.

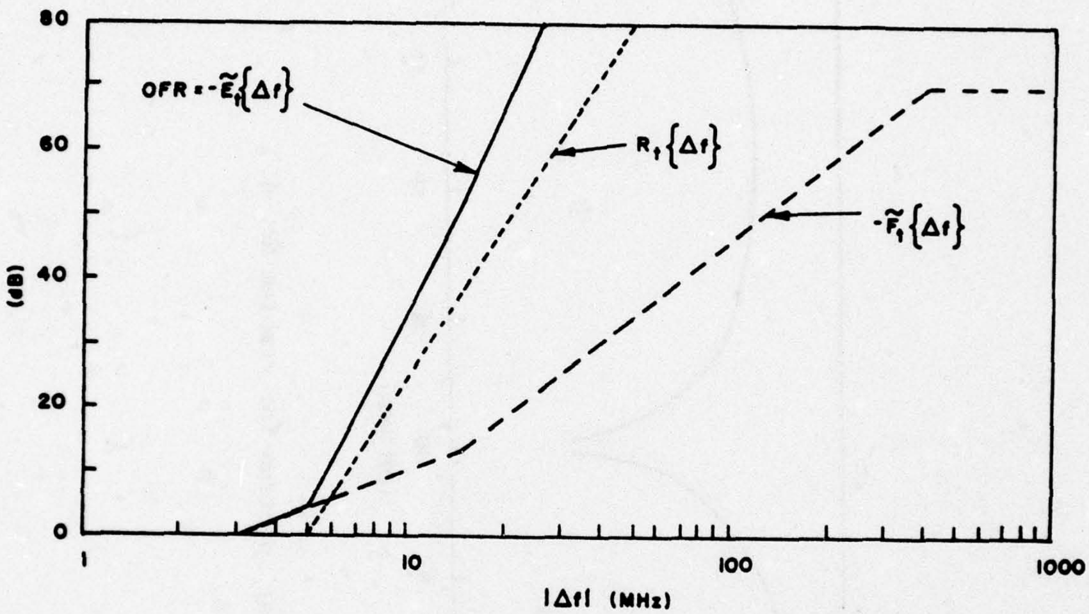

Figure G-3. Plot of OFR for Problem No. 3.

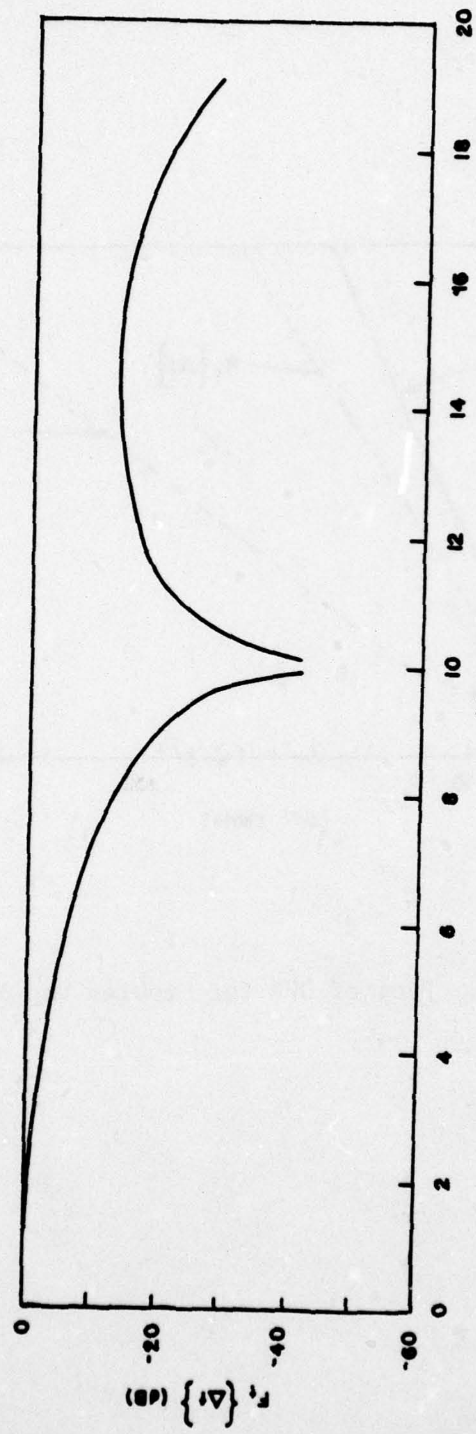

Figure G-4. Plot of spectrum for Problem No. 4.

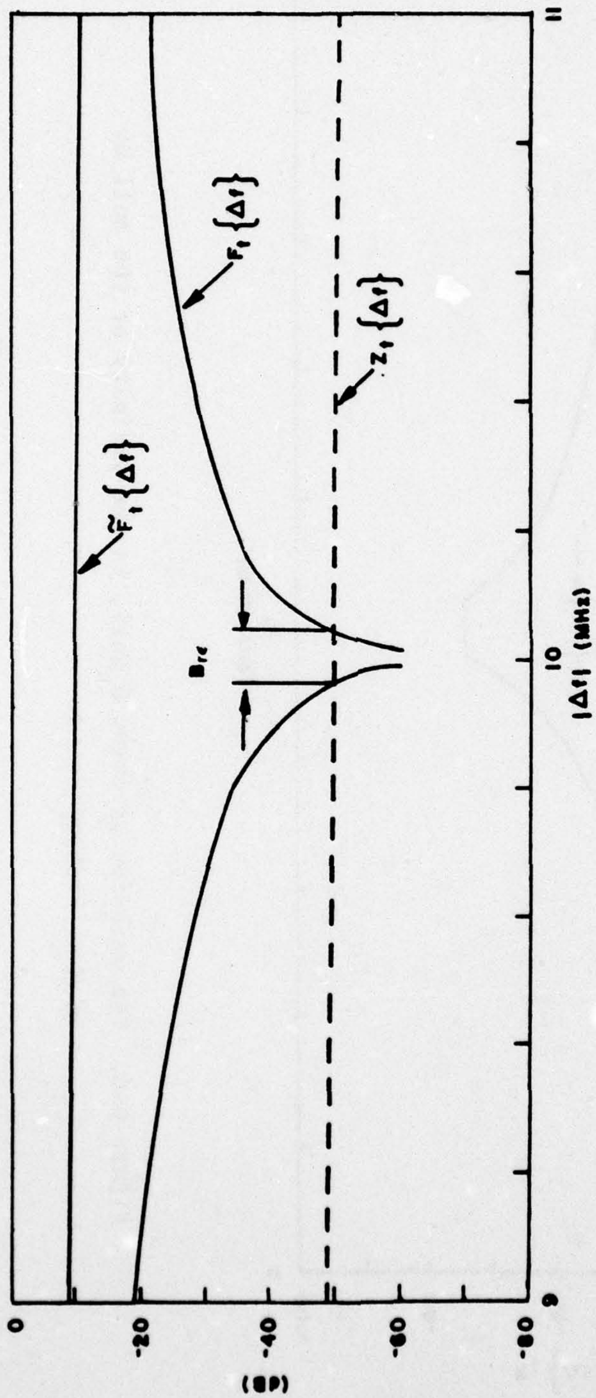

Figure G-5. Plots of spectral function in the vicinity of the null at 10 MHz, Problem No. 4.

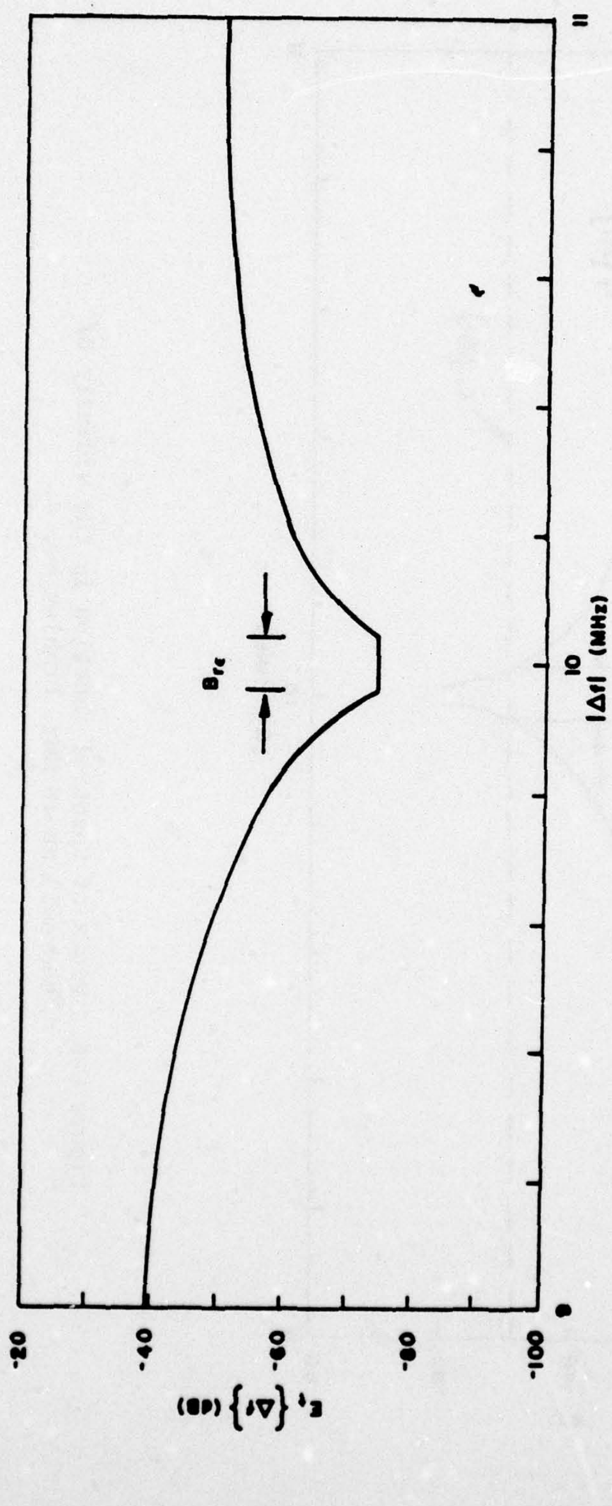

Figure G-6. The emission spectrum $E_t\{\Delta f\}$, in the vicinity of the null at 10 MHz, Problem No. 4.

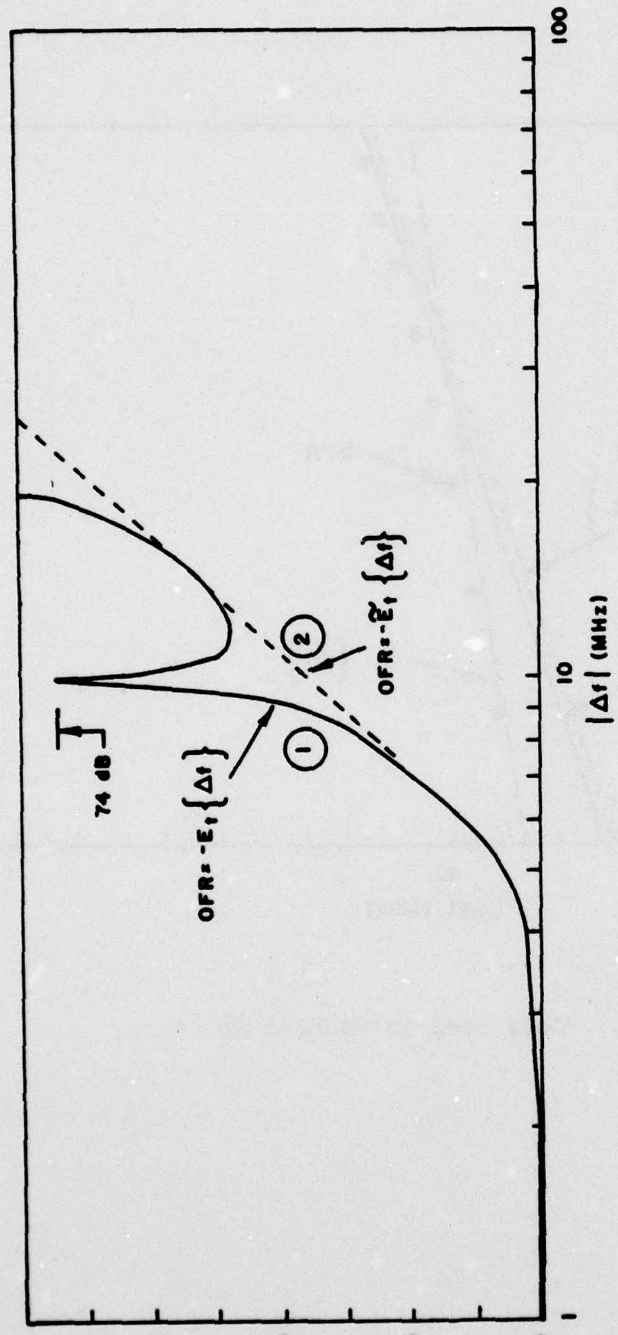

Figure G-7. Plot of OFR for Problem No. 4; Curve 1 using spectrum $E_t\{\Delta f\}$, Curve 2 using spectral bounds $\tilde{E}_t\{\Delta f\}$.

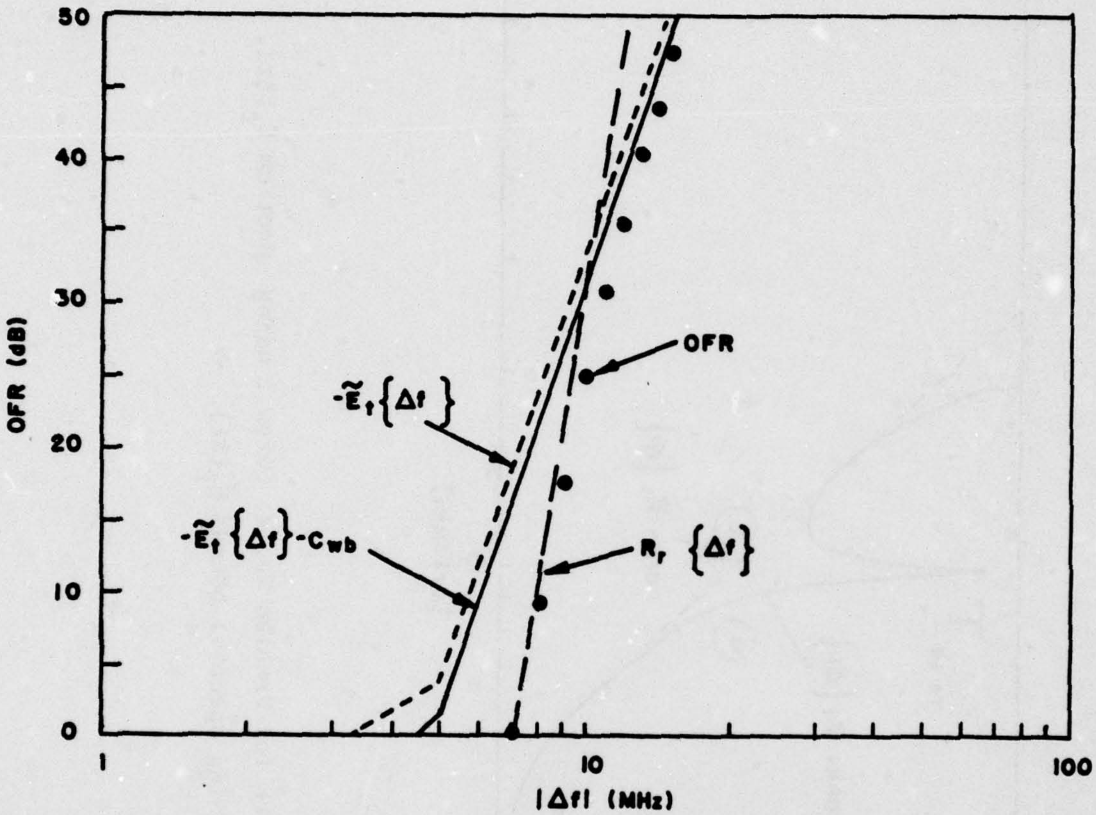

Figure G-8. Plots used in Problem No. 6.

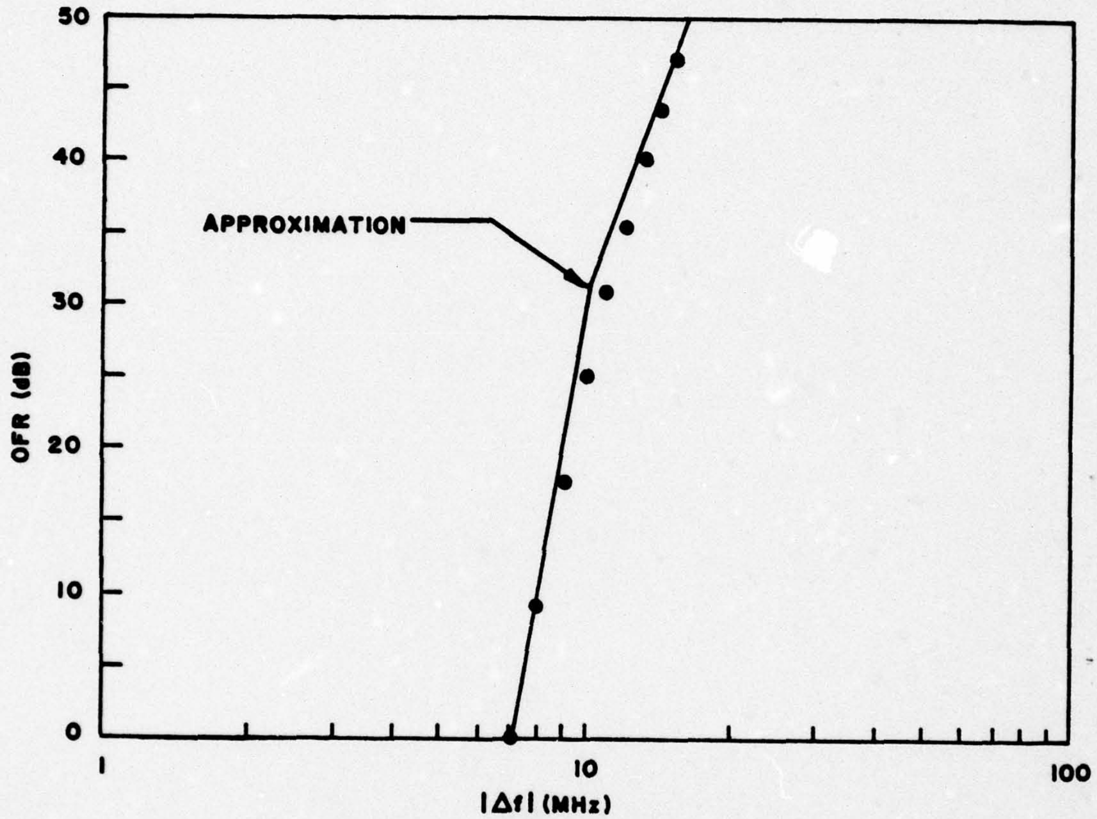

Figure G-9. Comparison of OFR calculated using the approximations (Equation G-5) and the more accurate method (Equation G-4), Problem No. 6.

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